Visualisierung räumlicher Mechanismen in unterschiedlichen Umgebungen

A. Gfrerrer

35. Fortbildungstagung für Geometrie
Strobl, 04. bis 06. September 2014

Mechanismen

Visualisierung/Animation von Mechanismen in einem CAD-Paket

Visualisierung/Animation von Mechanismen in einem Computer Algebra System

... oder lieber selbst programmieren?

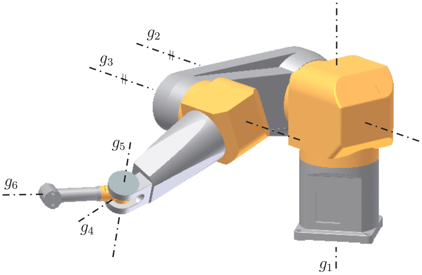
Beispiele

Mechanismen

Was ist ein Mechanismus?

Was ist ein Mechanismus?

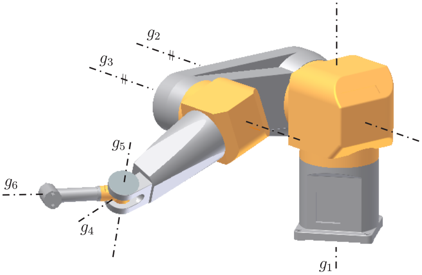
n Systeme Σ_i, $i = 0, \ldots, n-1$ (Starrkörper)



Was ist ein Mechanismus?

n Systeme Σ_i, $i = 0, \ldots, n-1$ (Starrkörper)

$n-1$ einparametrige Bewegungen $\Sigma_i \setminus \Sigma_0$, $i = 1, \ldots, n-1$

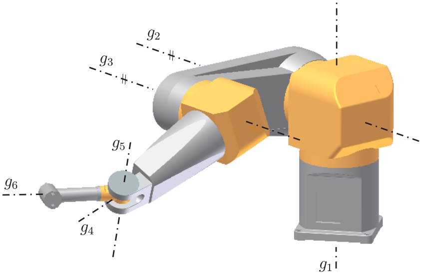


Was ist ein Mechanismus?

n Systeme Σ_i, $i = 0, \ldots, n-1$ (Starrkörper)

$n-1$ einparametrige Bewegungen $\Sigma_i \setminus \Sigma_0$, $i = 1, \ldots, n-1$

$$\Sigma_i \setminus \Sigma_0 : \mathbf{X}_0 = \mathbf{R}_x(u_1) \cdot \mathbf{C}_1 \cdot \ldots \cdot \mathbf{C}_{i-1} \cdot \mathbf{R}_x(u_i) \cdot \mathbf{X}_i$$



Was ist ein Mechanismus?

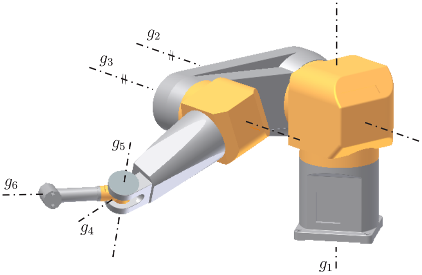
n Systeme Σ_i, $i = 0, \dots, n-1$ (Starrkörper)

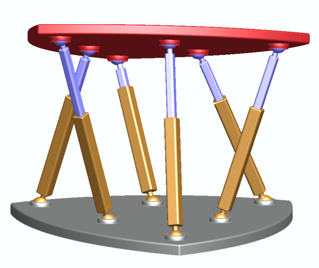
$n-1$ einparametrige Bewegungen $\Sigma_i \setminus \Sigma_0$, $i = 1, \dots, n-1$

$$\Sigma_i \setminus \Sigma_0 : \mathbf{X}_0 = \mathbf{R}_x(u_1) \cdot \mathbf{C}_1 \cdot \ldots \cdot \mathbf{C}_{i-1} \cdot \mathbf{R}_x(u_i) \cdot \mathbf{X}_i$$

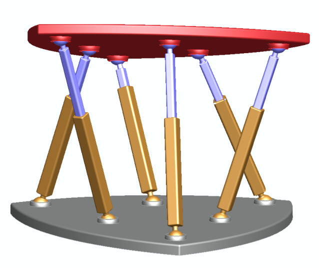
$$\mathbf{C}_j := \begin{bmatrix} 1 & 0 & 0 & 0 \\ d_j & \cos\alpha_j & -\sin\alpha_j & 0 \\ 0 & \sin\alpha_j & \cos\alpha_j & 0 \\ a_j & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_x(u_j) := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos u_j & -\sin u_j \\ 0 & 0 & \sin u_j & \cos u_j \end{bmatrix}$$

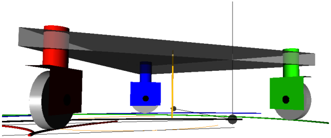




Stewart-Gough platform



Stewart-Gough platform



Omnidirektionaler mobiler Roboter

Visualisierung/Animation von Mechanismen in einem CAD-Paket

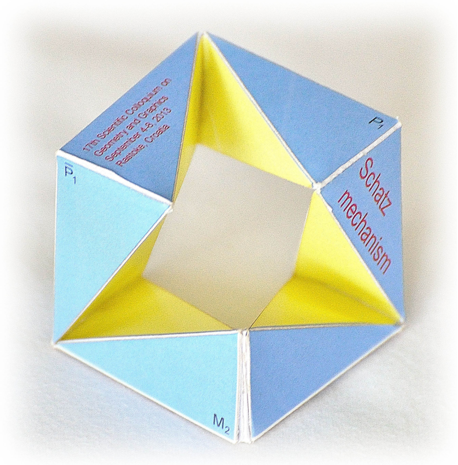
Visualisierung/Animation von Mechanismen in einem Computer Algebra System

... oder die Dinge doch lieber selbst programmieren?[1]

[1] der aufwändigste aber auch zufriedenstellendste Weg

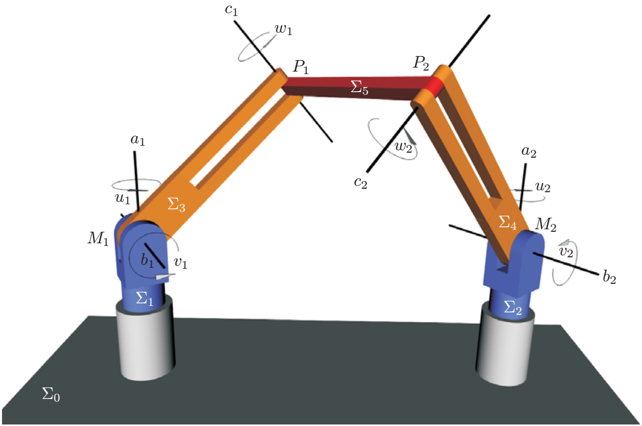
Beispiel: Schatz-Mechanismus

Variante 1

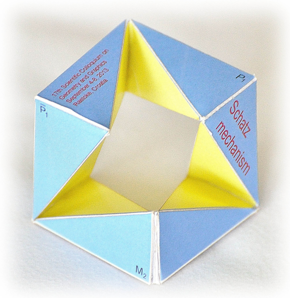


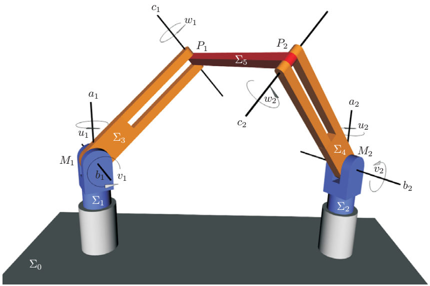
Beispiel: Der Schatz-Mechanismus

Variante 2



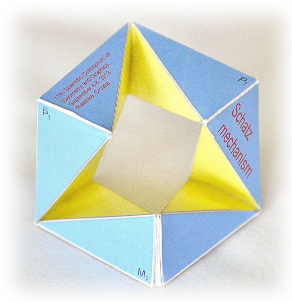
Dreiecke $P_1, \overline{P}_1, M_2$ und $P_2, \overline{P}_2, M_1$:

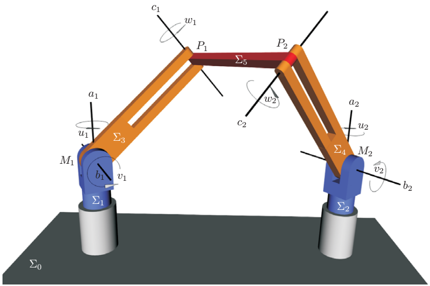


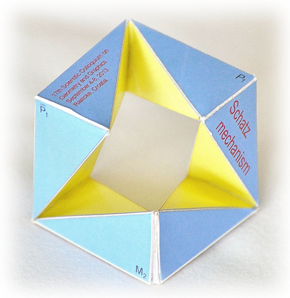


Dreiecke $P_1, \overline{P}_1, M_2$ und $P_2, \overline{P}_2, M_1$:

- gleichseitig

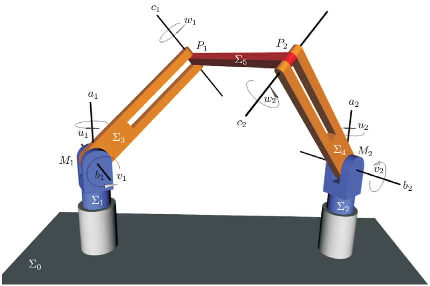






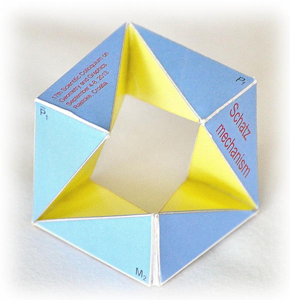
Dreiecke $P_1, \overline{P}_1, M_2$ und $P_2, \overline{P}_2, M_1$:

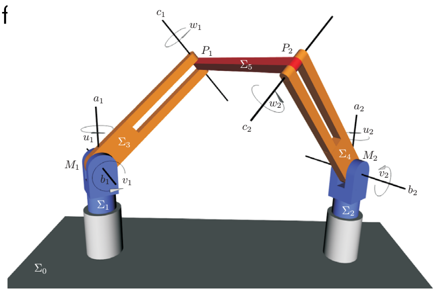
- gleichseitig
- zentrisch ähnlich mit dem Mittelpunkt C des Würfels als Ähnlichkeitszentrum



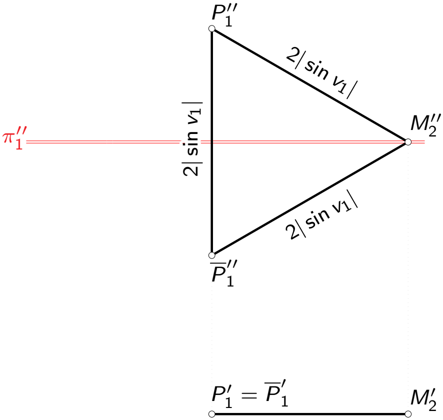
Dreiecke $P_1, \overline{P}_1, M_2$ und $P_2, \overline{P}_2, M_1$:

- gleichseitig
- zentrisch ähnlich mit dem Mittelpunkt C des Würfels als Ähnlichkeitszentrum
- Schwerpunkte der Dreiecke liegen auf einer gemeinsamen Normalen ihrer parallelen Trägerebenen

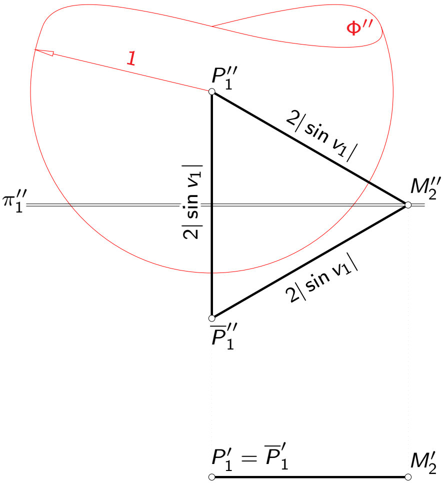




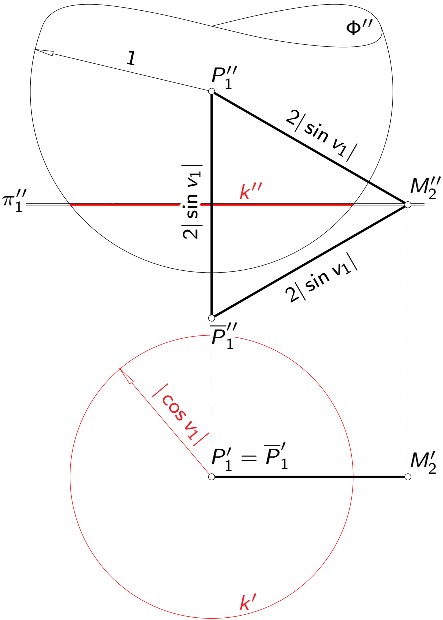
Die Darstellende Geometrie
hilft uns hier!



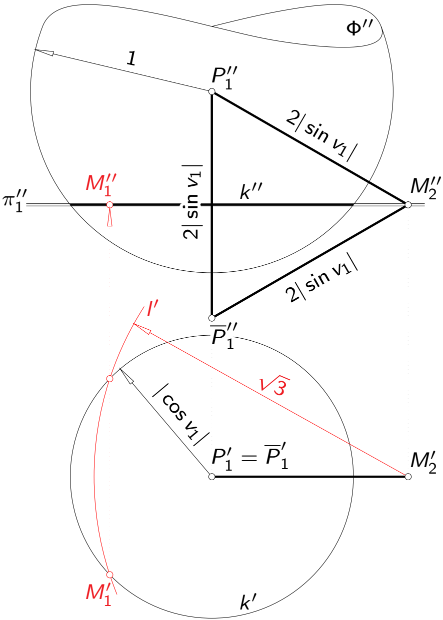
Die Darstellende Geometrie
hilft uns hier!



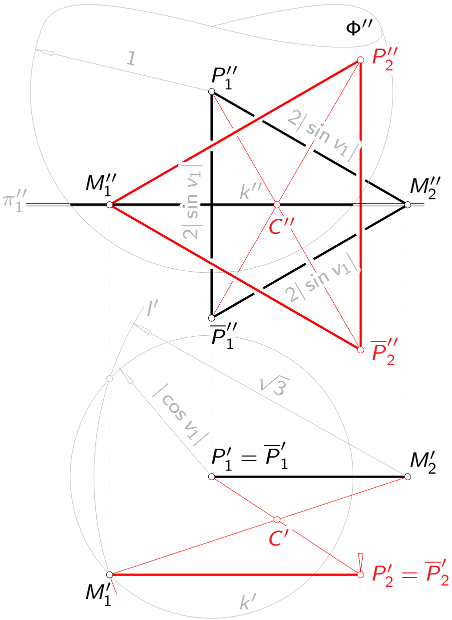
Die Darstellende Geometrie
hilft uns hier!



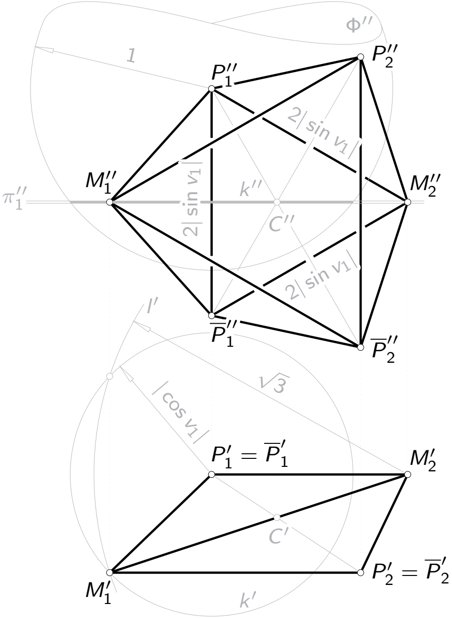
Die Darstellende Geometrie
hilft uns hier!



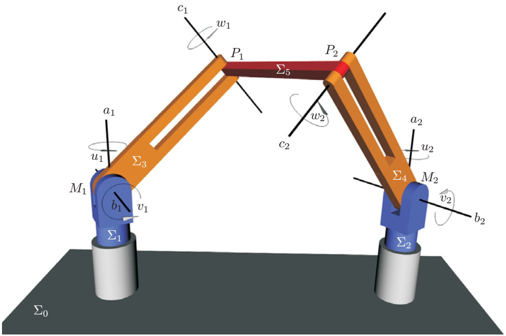
Die Darstellende Geometrie
hilft uns hier!



Die Darstellende Geometrie
hilft uns hier!

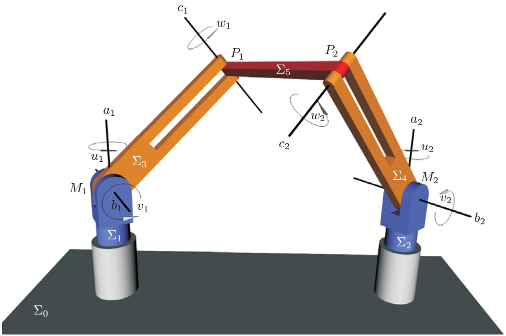


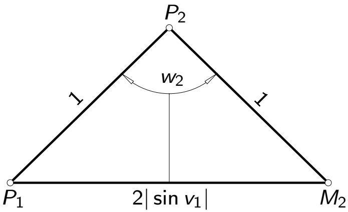
Winkelrelationen



Winkelrelationen

$$w_2 = 2v_1$$

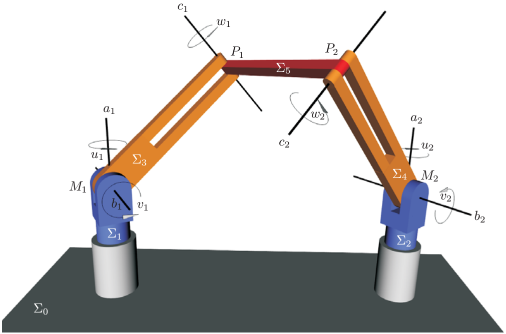


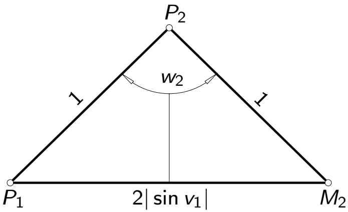


Winkelrelationen

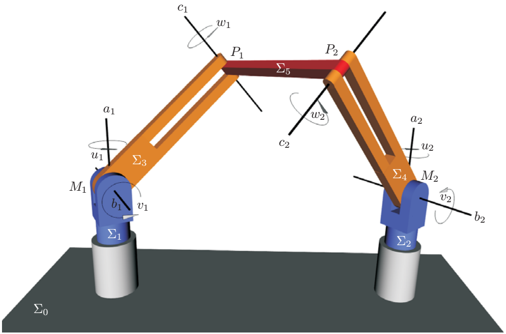
$$w_2 = 2v_1$$

$$w_1 = 2v_2$$

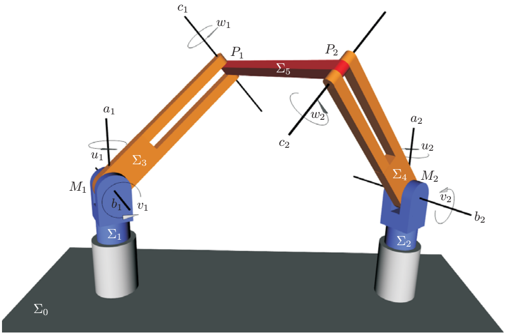


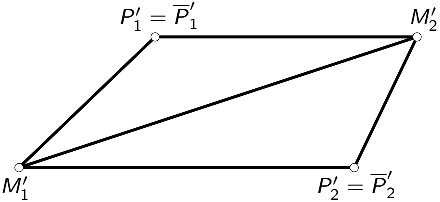


Winkelrelationen

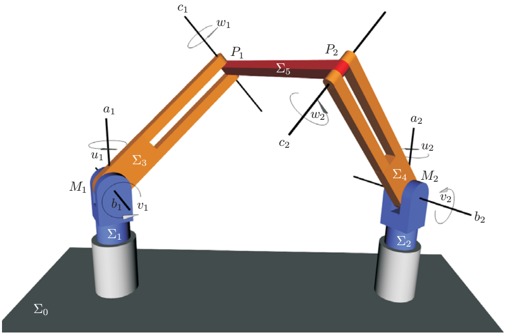


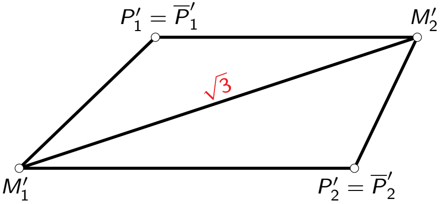
Winkelrelationen



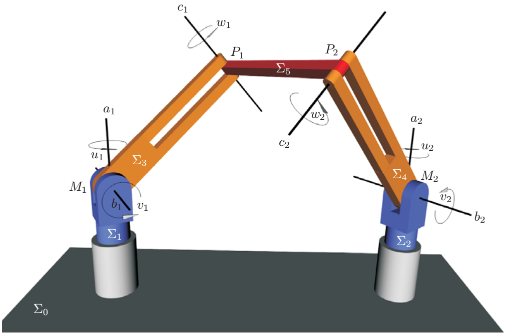


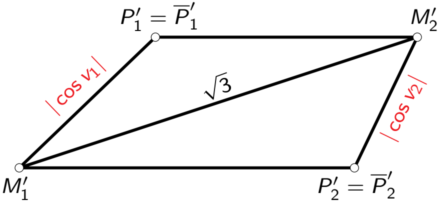
Winkelrelationen



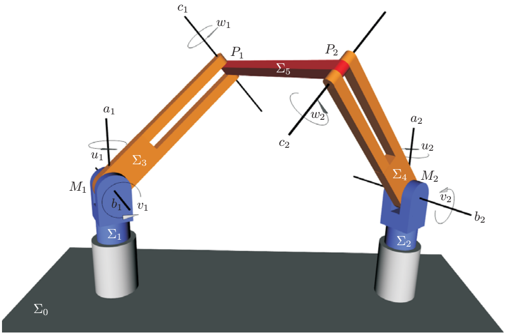


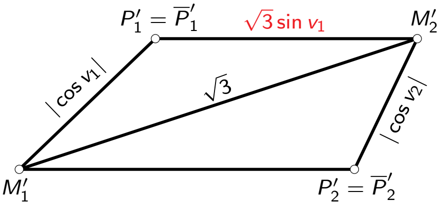
Winkelrelationen



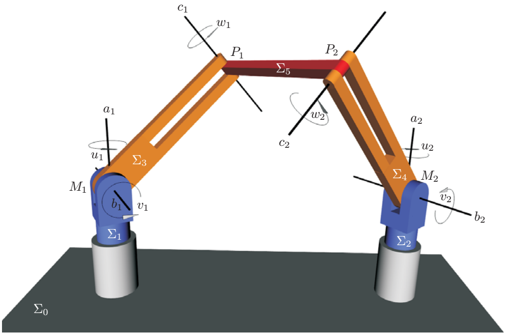


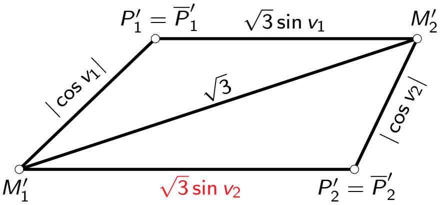
Winkelrelationen



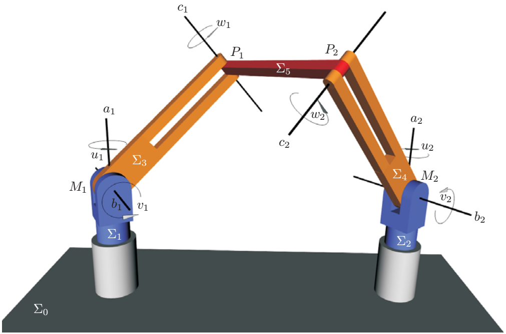


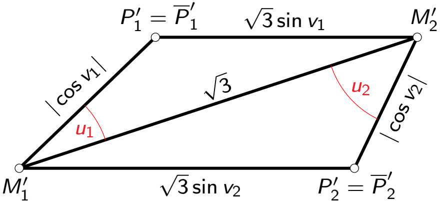
Winkelrelationen

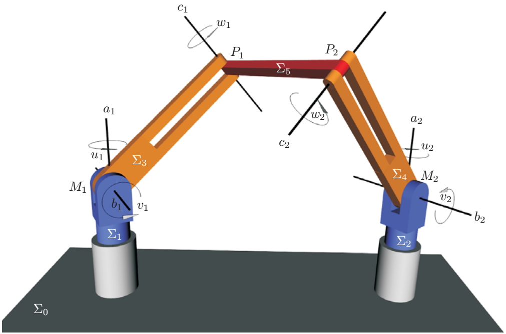




Winkelrelationen

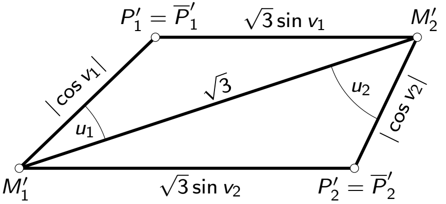




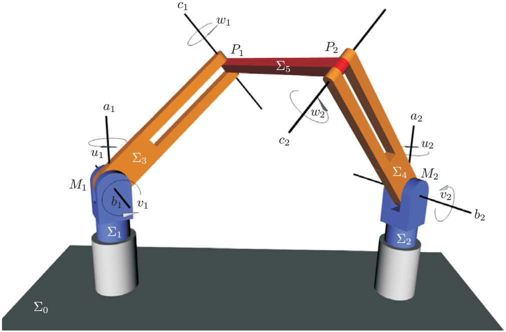


Winkelrelationen

mittels Kosinussatz:

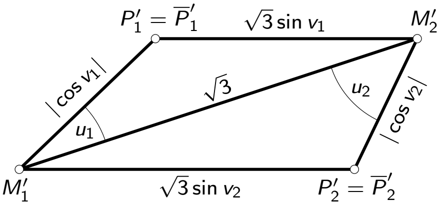


Winkelrelationen

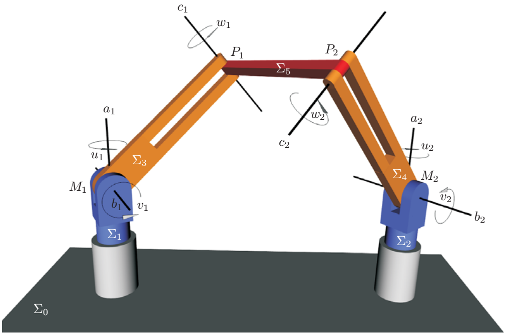


mittels Kosinussatz:

$$\cos v_1 = \pm\frac{\sqrt{3}}{2}\cos u_1$$

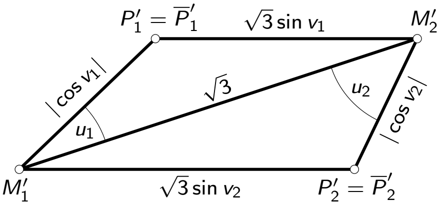


Winkelrelationen



mittels Kosinussatz:

$$\cos v_1 = \pm\frac{\sqrt{3}}{2}\cos u_1$$
$$\sin v_1 = \pm\frac{1}{2}\sqrt{4 - 3\cos^2 u_1}$$

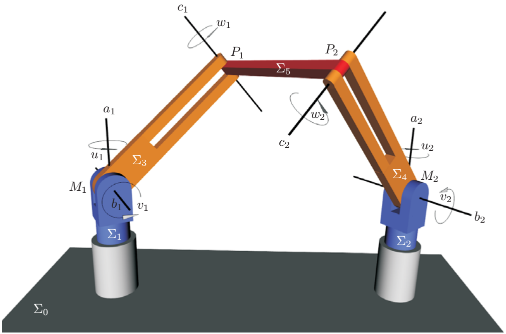


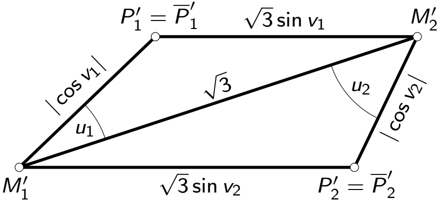
Winkelrelationen

mittels Kosinussatz:

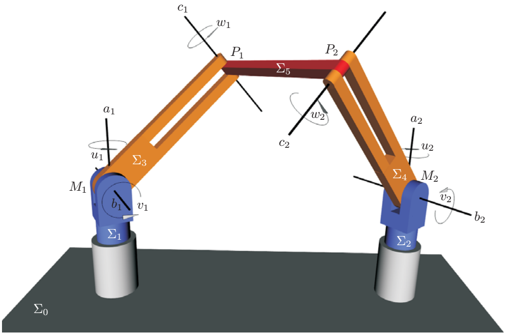
$$\cos v_1 = \pm\frac{\sqrt{3}}{2}\cos u_1$$
$$\sin v_1 = \pm\frac{1}{2}\sqrt{4 - 3\cos^2 u_1}$$

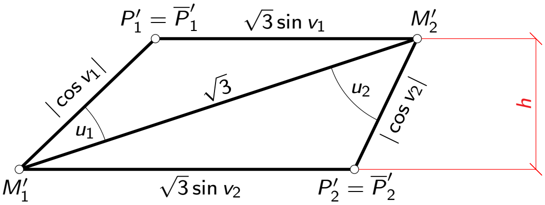
$$\cos v_2 = \pm\frac{\sqrt{3}}{2}\cos u_2$$
$$\sin v_2 = \pm\frac{1}{2}\sqrt{4 - 3\cos^2 u_2}$$





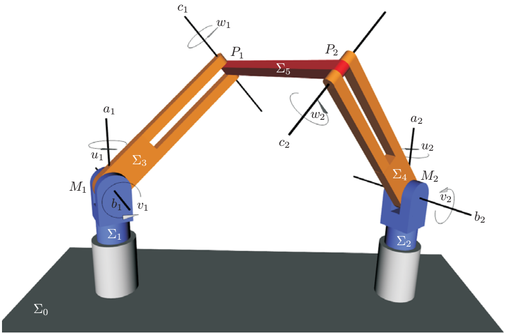
Winkelrelationen

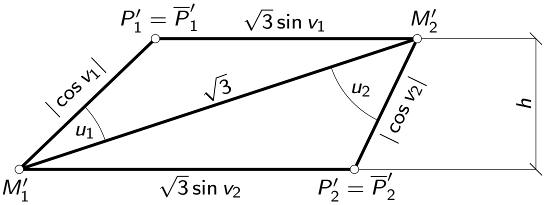




Winkelrelationen

$$\frac{\sqrt{3}|\cos v_1||\sin u_1|}{2} = \frac{\sqrt{3}|\sin v_1|h}{2}$$

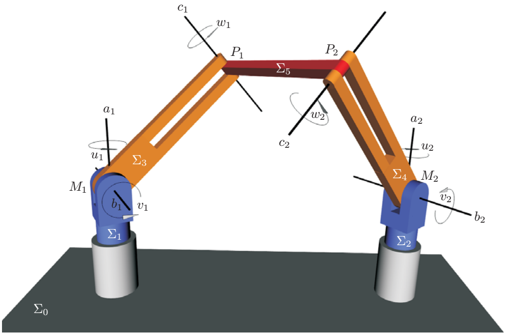


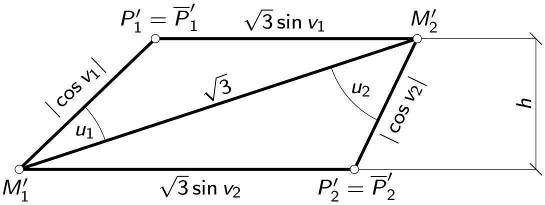


Winkelrelationen

$$\frac{\sqrt{3}|\cos v_1||\sin u_1|}{2} = \frac{\sqrt{3}|\sin v_1|h}{2}$$

$$\frac{\sqrt{3}|\cos v_2||\sin u_2|}{2} = \frac{\sqrt{3}|\sin v_2|h}{2}$$



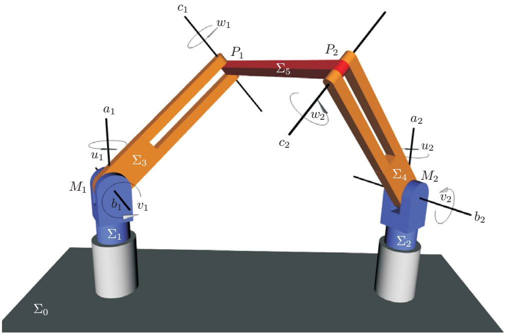


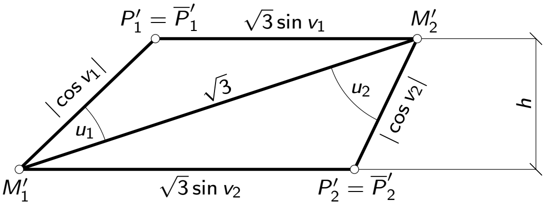
Winkelrelationen

$$\frac{\sqrt{3}|\cos v_1||\sin u_1|}{2} = \frac{\sqrt{3}|\sin v_1|h}{2}$$

$$\frac{\sqrt{3}|\cos v_2||\sin u_2|}{2} = \frac{\sqrt{3}|\sin v_2|h}{2}$$

$$\left|\frac{\sin u_1}{\sin u_2}\right| = \left|\frac{\sin v_1}{\cos v_1} \cdot \frac{\cos v_2}{\sin v_2}\right|$$





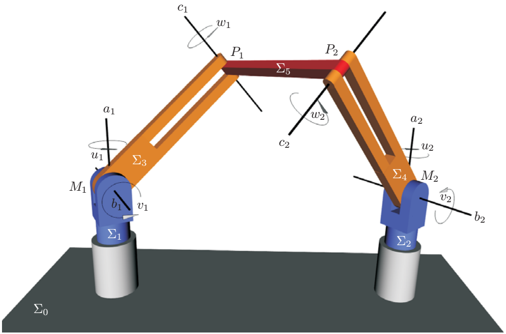
Winkelrelationen

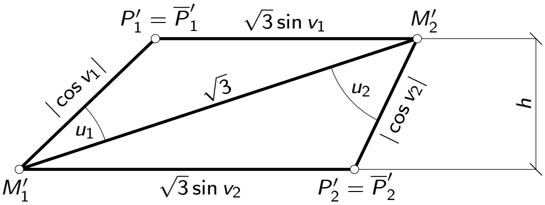
$$\frac{\sqrt{3}|\cos v_1||\sin u_1|}{2} = \frac{\sqrt{3}|\sin v_1|h}{2}$$

$$\frac{\sqrt{3}|\cos v_2||\sin u_2|}{2} = \frac{\sqrt{3}|\sin v_2|h}{2}$$

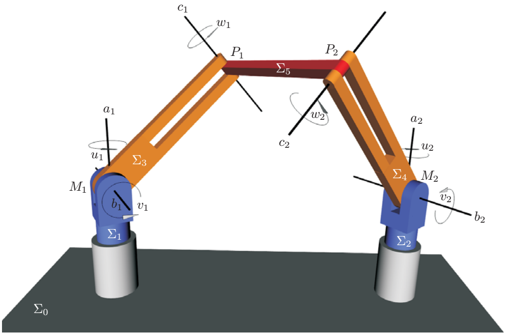
$$\left|\frac{\sin u_1}{\sin u_2}\right| = \left|\frac{\sin v_1}{\cos v_1} \cdot \frac{\cos v_2}{\sin v_2}\right|$$

$$\cos u_2 = \pm\frac{2\sin u_1}{\sqrt{4-3\cos^2 u_1}}$$





Winkelrelationen



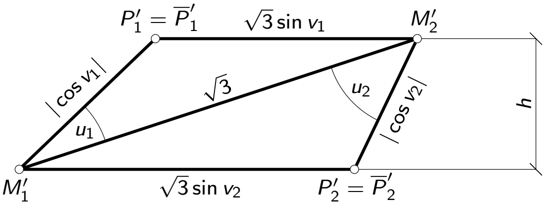
$$\frac{\sqrt{3}|\cos v_1||\sin u_1|}{2} = \frac{\sqrt{3}|\sin v_1|h}{2}$$

$$\frac{\sqrt{3}|\cos v_2||\sin u_2|}{2} = \frac{\sqrt{3}|\sin v_2|h}{2}$$

$$\left|\frac{\sin u_1}{\sin u_2}\right| = \left|\frac{\sin v_1}{\cos v_1} \cdot \frac{\cos v_2}{\sin v_2}\right|$$

$$\cos u_2 = \pm\frac{2\sin u_1}{\sqrt{4-3\cos^2 u_1}}$$

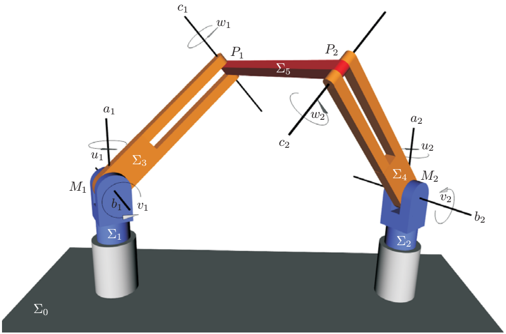
$$\sin u_2 = \pm\frac{\cos u_1}{\sqrt{4-3\cos^2 u_1}}$$

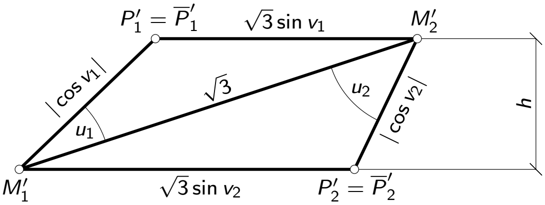


Winkelrelationen

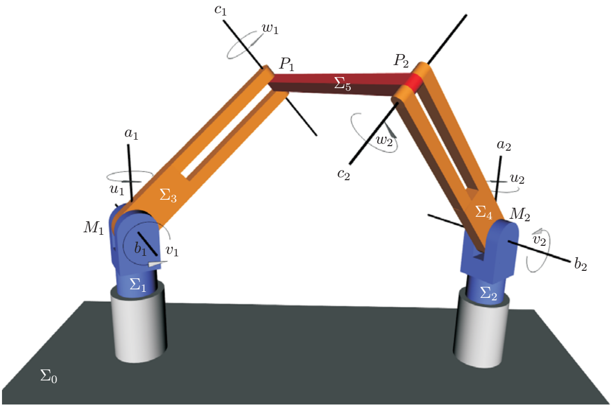
$$\cos w_1 = \frac{3\sin^2 u_1 - 1}{4 - 3\cos^2 u_1}$$

$$\sin w_1 = \pm\frac{2\sqrt{3}\sin u_1}{4 - 3\cos^2 u_1}$$





Winkelrelationen



Mit Hilfe dieser Winkelabhängigkeiten lassen sich alle weiteren leicht bestimmen.

$\Longrightarrow$

Parameterdarstellungen der Bewegungen Σ_i / Σ_j.

Beispiele

Amon, H., Overconstrained mechanisms derived from planar chains of linkages, Diploma Thesis TU Graz, 2013.

Zsombor-Murray, P.J., Gfrerrer A., Robotrac Mobile 6R Closed Chain, Proceedings of CSME Forum, Queen's University, Kingston, Ontario, 1–4, 2002.

Li, Z., Schicho, J., Three Types of Parallel 6R Linkages, Proceedings of the 6th International Workshop on Computational Kinematics (CK2013), Federico Thomas, Alba Perez Gracia (editors), , Springer, 111–119, 2013.

Stachel, H., Ein bewegliches Tetraederpaar. (A movable pair of tetrahedra), Elem. Math. **43**, 3, 65–75, 1988.

Stolzlechner, J., Visualization of spatial mechanisms in standard 3D formats, Diploma Thesis TU Graz, 2013.

Danke für die Aufmerksamkeit!*)

*) und danke an das SMM:[2] Romana Lührmann Sebastian Stubenrauch Johanna Gfrerrer

[2] Schatz Mechanism Manufactory