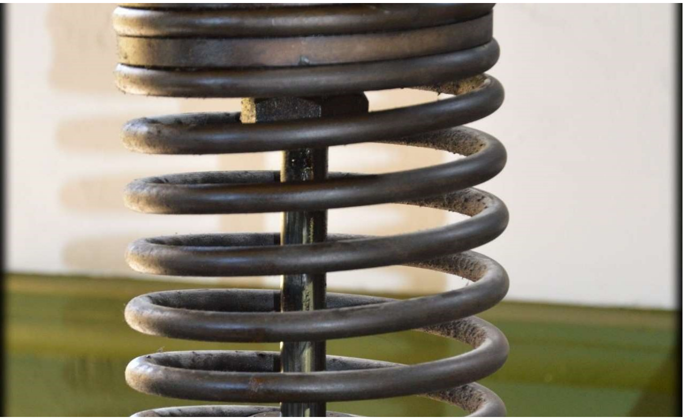
Über Spiralungen

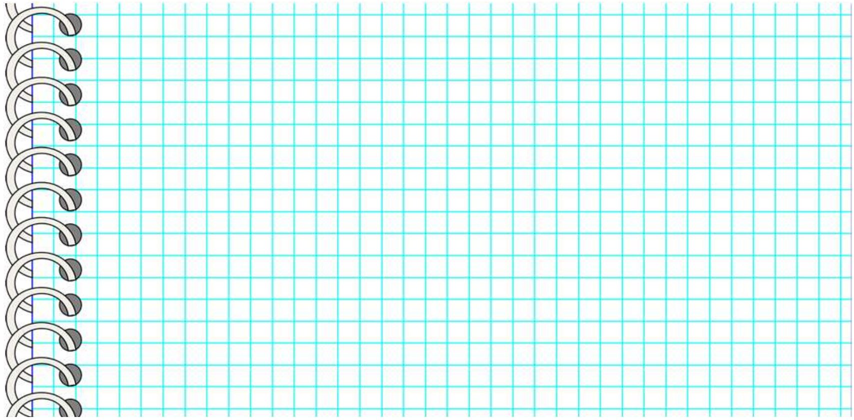
Bachelorarbeit an der TU-Graz

betreut von Prof. Anton Gfrerrer

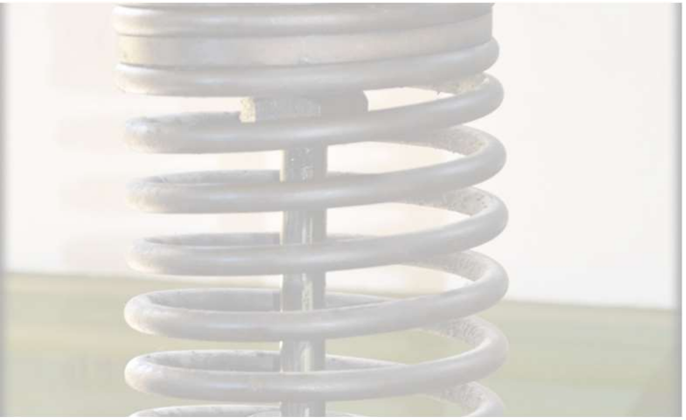
Kristina M. Ranegger

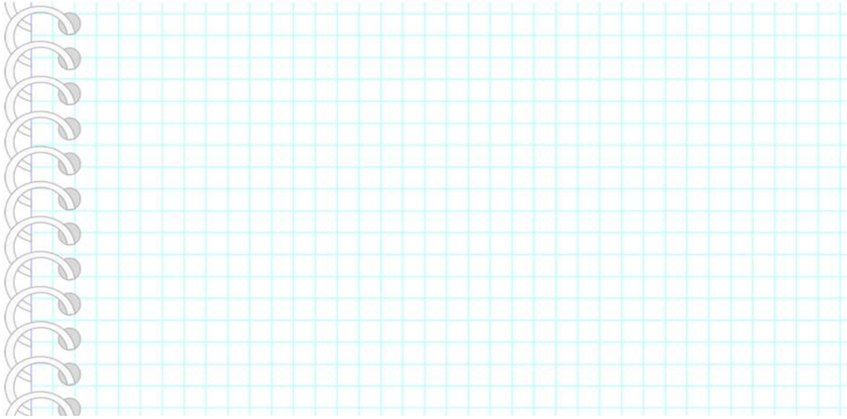

FEUER
NOTRUF





SCHRAUBEN ≠SPIRALEN





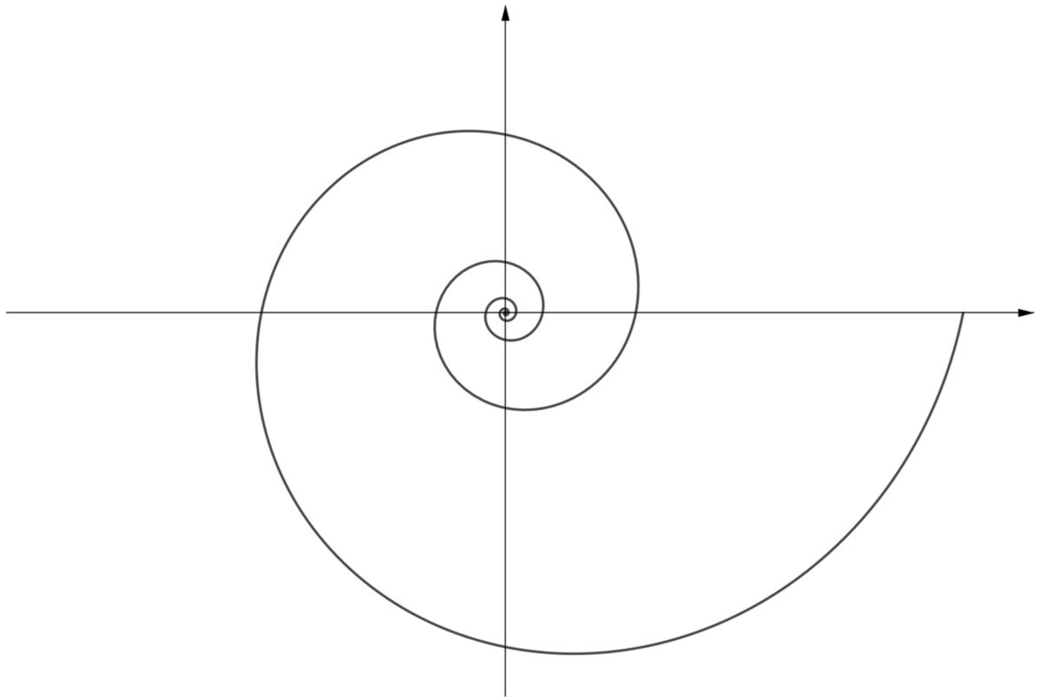
Spiralgalaxie

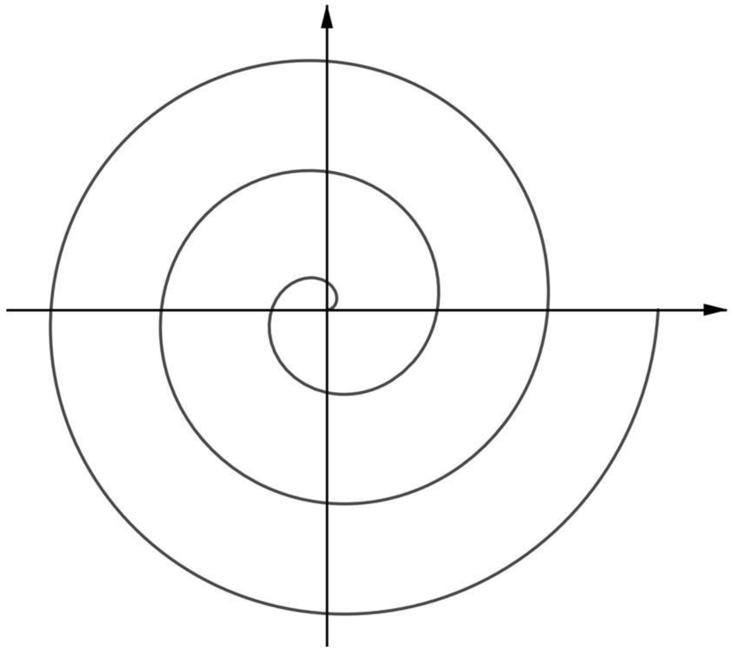

urtümlicher Ammonit (Marokko, vor ca. 390 Mio. Jahren)



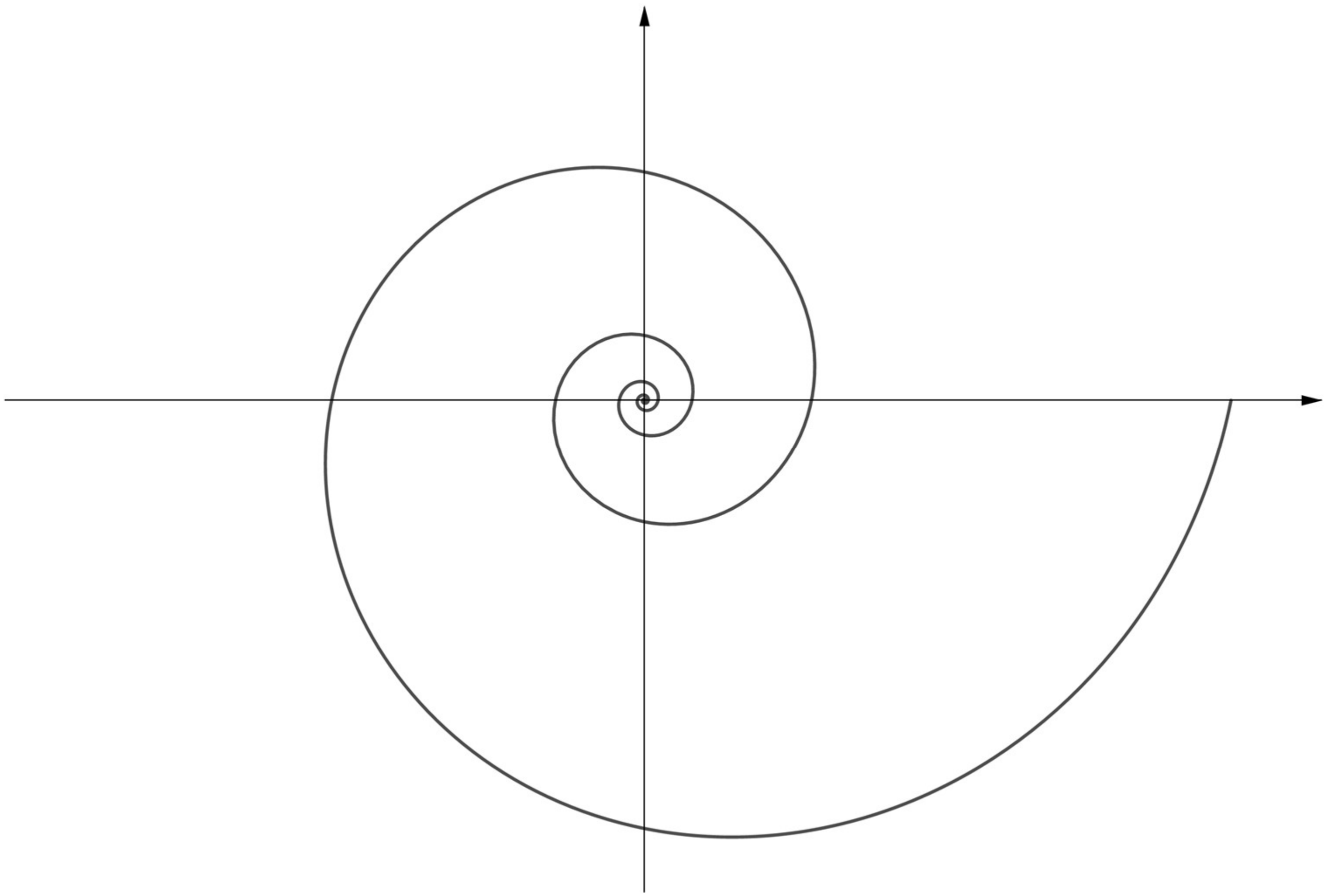


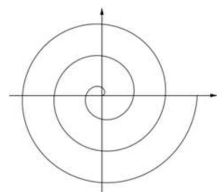
ebene Spiralungen



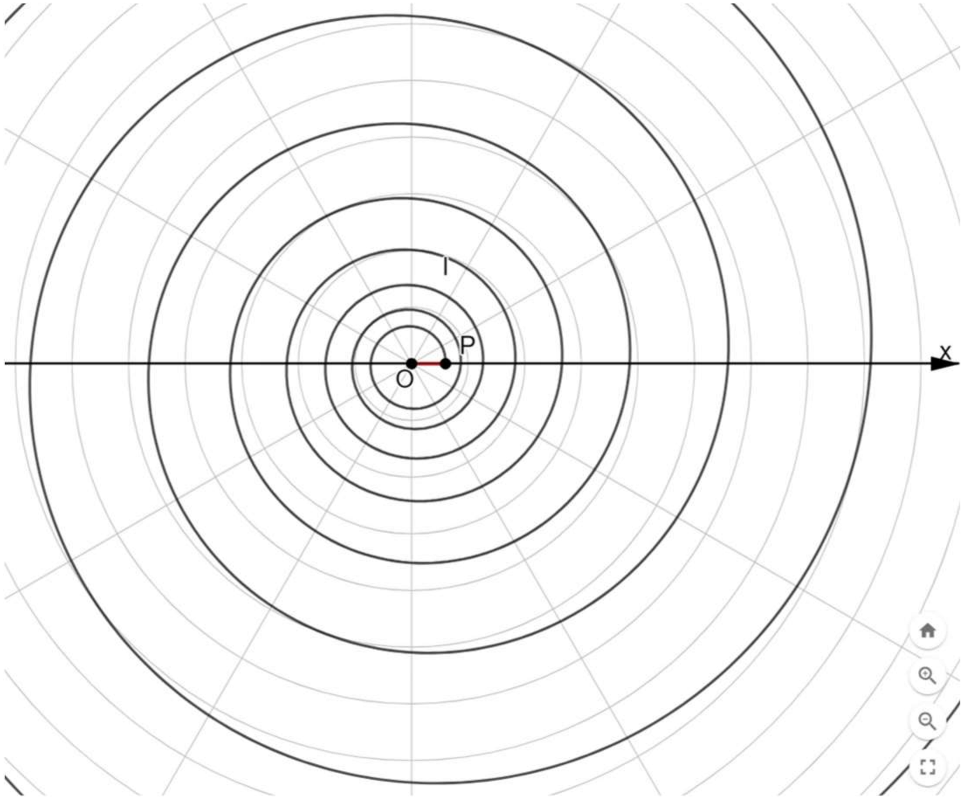


logarithmische Spirale





logarithmische Spirale



Satz 2. Alle Punkte $P_n\,(r_n, \varphi_n)$ mit $r_n = r_0 \cdot s^n$, $\varphi_n = \varphi_0 + n \cdot \omega$ liegen auf einer Kurve mit der Parameterdarstellung $P\,(r, \varphi)$:

$$r = r_0 \cdot e^{p \cdot \sigma}, \;\; \varphi = \varphi_0 + \sigma \;, p \in \mathbb{R}, \, \sigma \in \mathbb{R} \tag{1}$$

mit $p = \dfrac{\ln(s)}{\omega}$.

Beweis. Setze $\sigma := n \cdot \omega$, dann folgt aus (1):

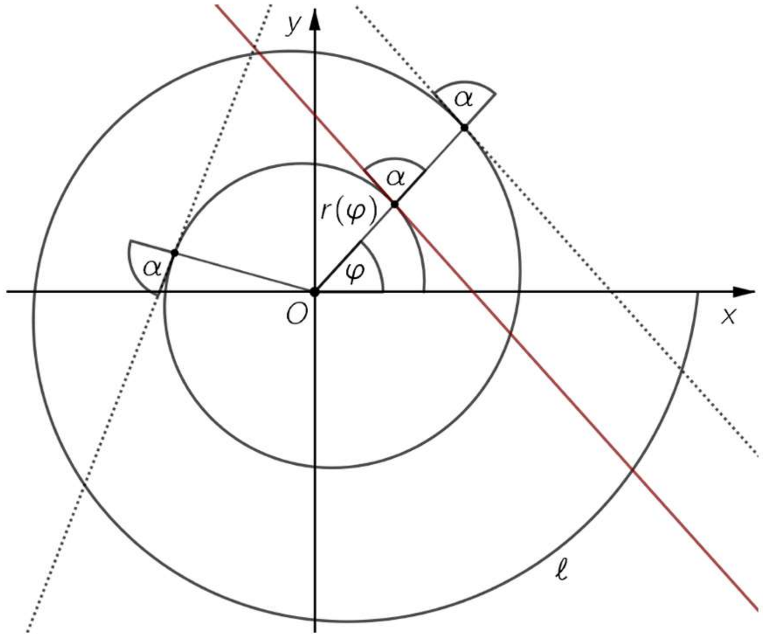
$$r = r_0 \cdot e^{p \cdot \sigma} = r_0 \cdot e^{\frac{\ln(s)}{\omega} \cdot n \cdot \omega} = r_0 \cdot \left(e^{\ln(s)}\right)^n = r_0 \cdot s^n, \tag{2}$$

$$\varphi = \varphi_0 + n \cdot \omega, \tag{3}$$

was genau die Polarkoordinaten von P_n sind. $\square$

$$P_0(r, \varphi), \; P_1(s \cdot r, \varphi + \omega), \; \ldots, \; P_n(s^n \cdot r, \varphi + n\omega)$$

logarithmische Spirale



Satz 3. Für den Kurswinkel α gilt: $\cot(\alpha) = p$.

Der Kurswinkel α hängt nur vom Spiralparameter p ab.
Das bedeutet, dass eine Streckung oder Drehung, eine logarithmische Spirale in eine dazu kongruente überführt.

„Eadem mutata resurgo“

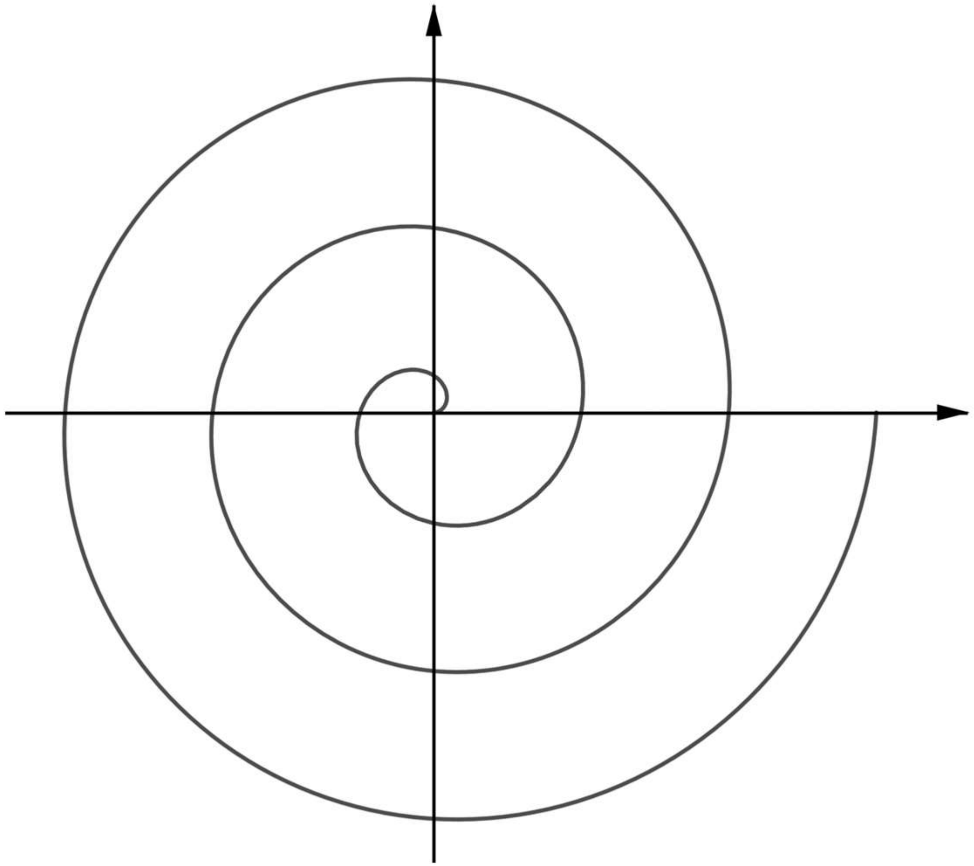
„verändert kehre ich als dieselbe wieder“

Inschrift Grabstein Jakob Bernoulli
(Schweiz, 1654 – 1705)

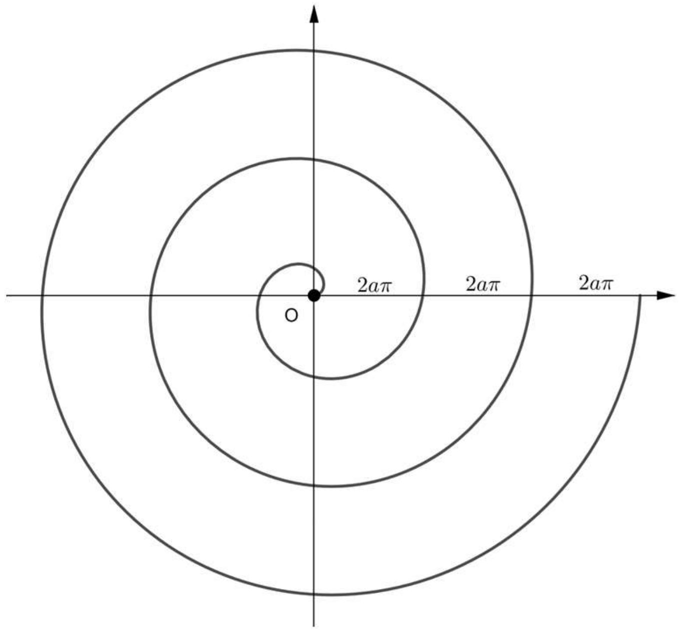
Sonar, T.: 3000 Jahre Analysis. Geschichte - Kulturen - Menschen. In: Alten, H. u. a. (Hg.): Vom Zählstein zum Computer. S. 422 (von Kahle), Heidelberg 2011.


EADEM MUTATA RESURGO

die archimedische Spirale



die archimedische Spirale



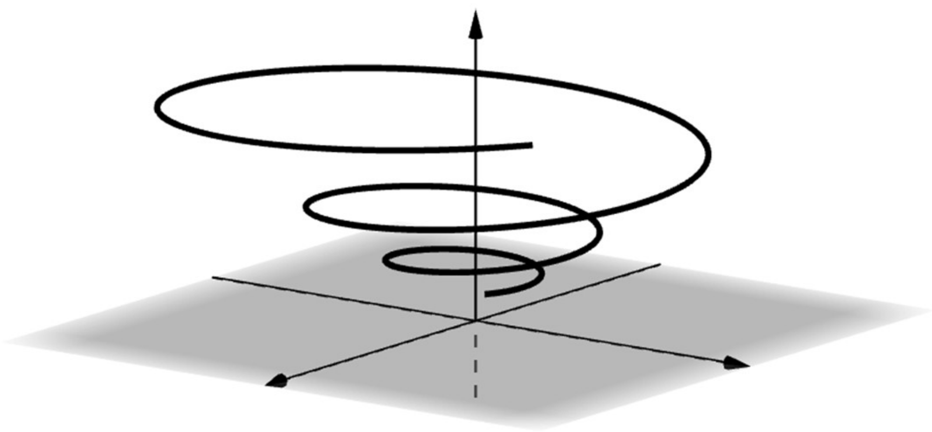
Definition 3. Eine Kurve ℓ mit der Gleichung in Polarkoordinaten $(r(\varphi), \varphi)$

$$r(\varphi) = a \cdot \varphi + b, \quad a = \text{const} \in \mathbb{R}^+, \ b = \text{const} \in \mathbb{R} \ \text{ und } \ \varphi \in \mathbb{R}^+ \cup \{0\}$$

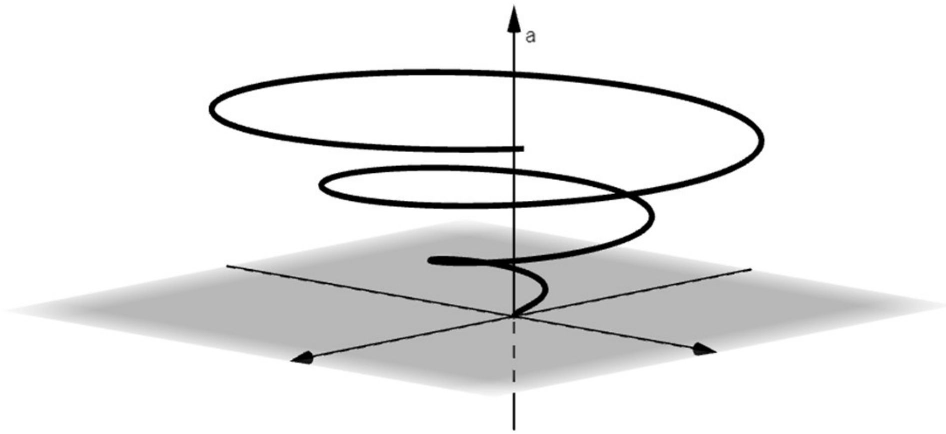
heißt **archimedische Spirale**.

Satz 4. Der Windungsabstand ist bei archimedischen Spiralen konstant.

räumliche Spiralungen



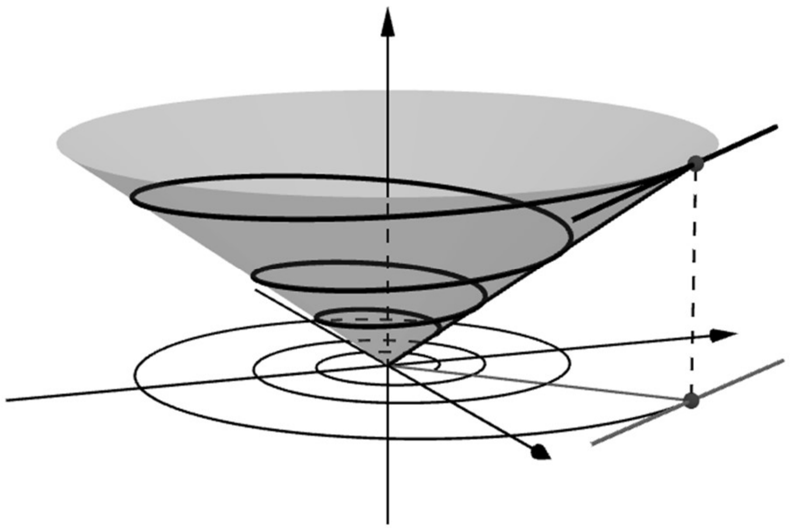
klassische Spirale



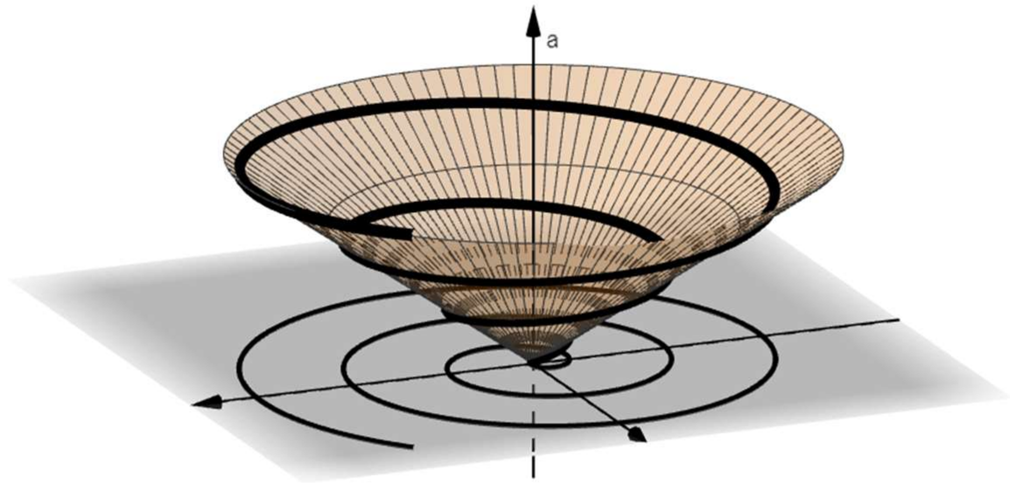
Helispirale

räumliche Spiralungen

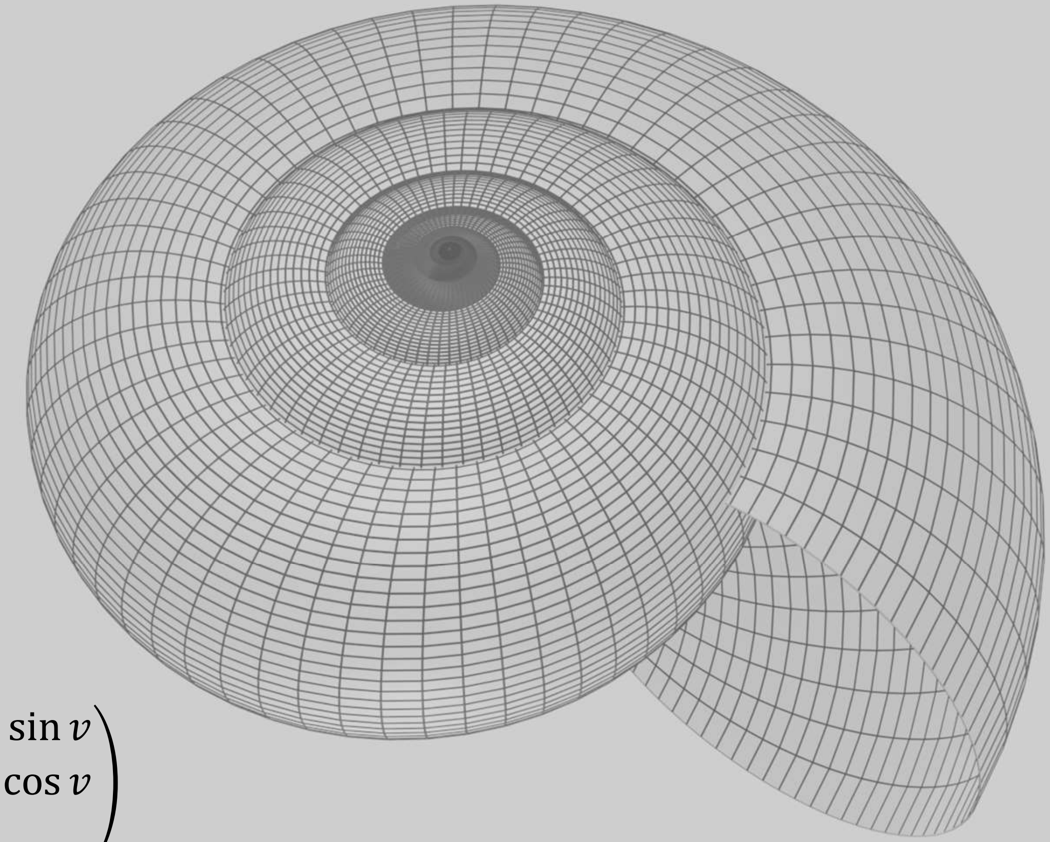
klassische Spirale



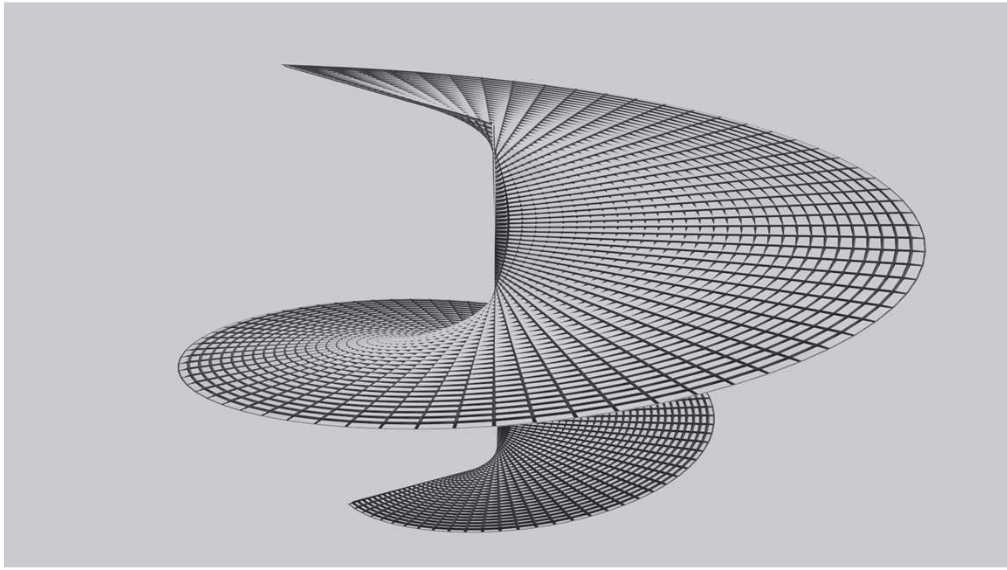
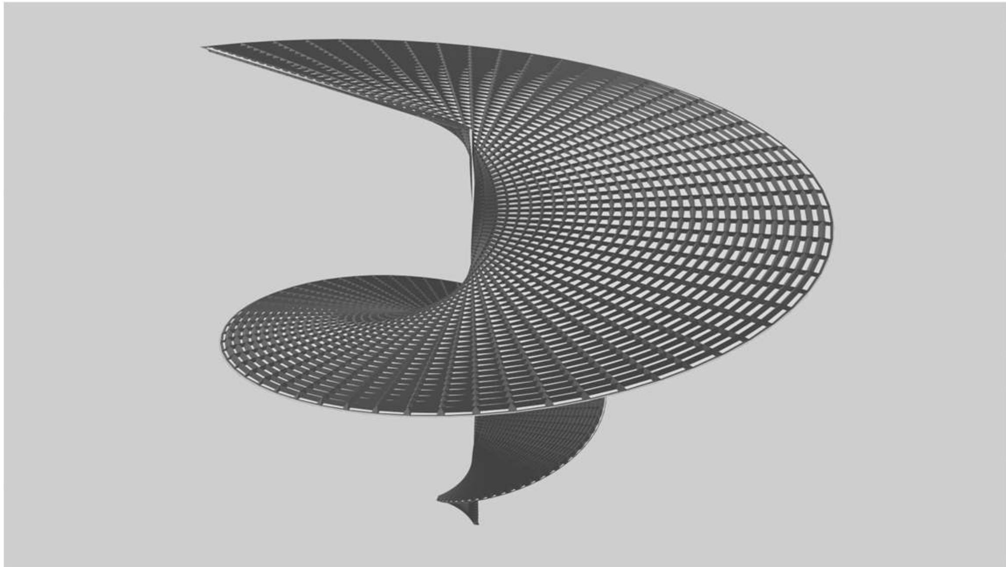
Helispirale



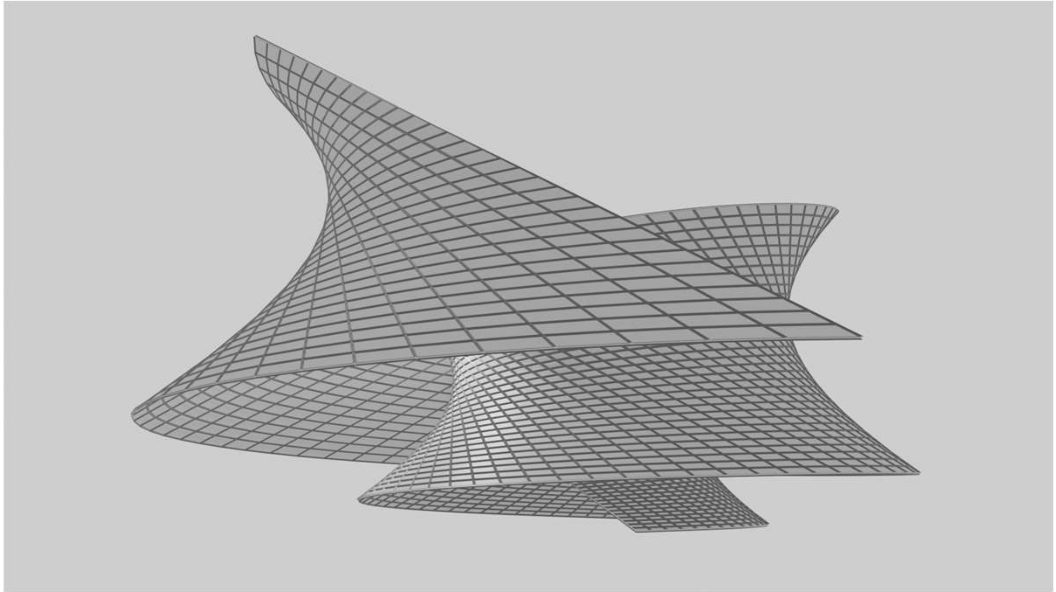
Spiralflächen

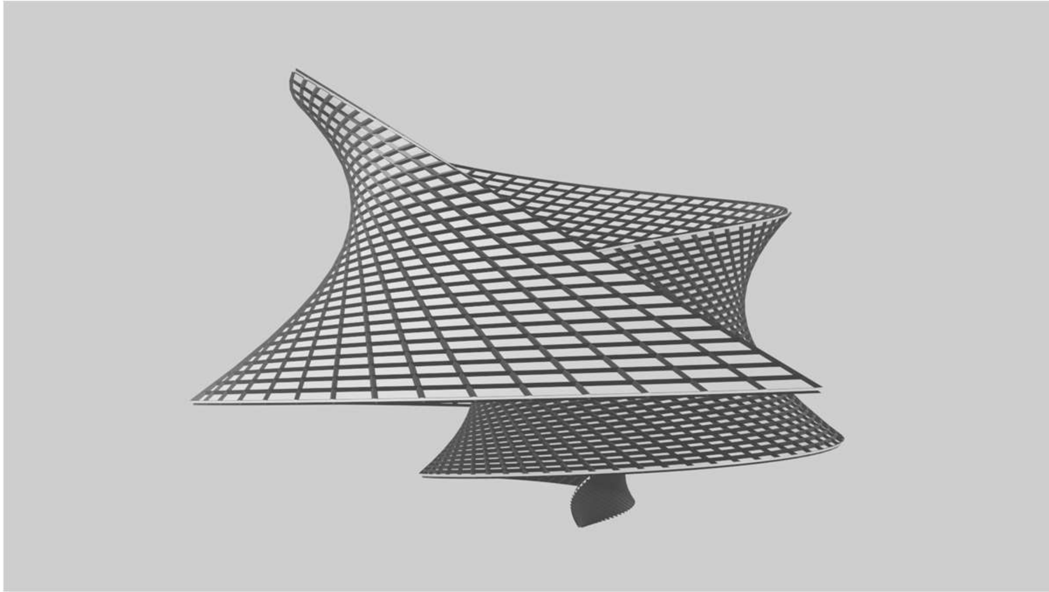


$$\boldsymbol{\phi}(u,v) = s(v) \cdot \begin{pmatrix} x_0(u) \cdot \cos v - y_0(u) \cdot \sin v \\ x_0(u) \cdot \sin v + y_0(u) \cdot \cos v \\ z_0(u) \end{pmatrix}$$

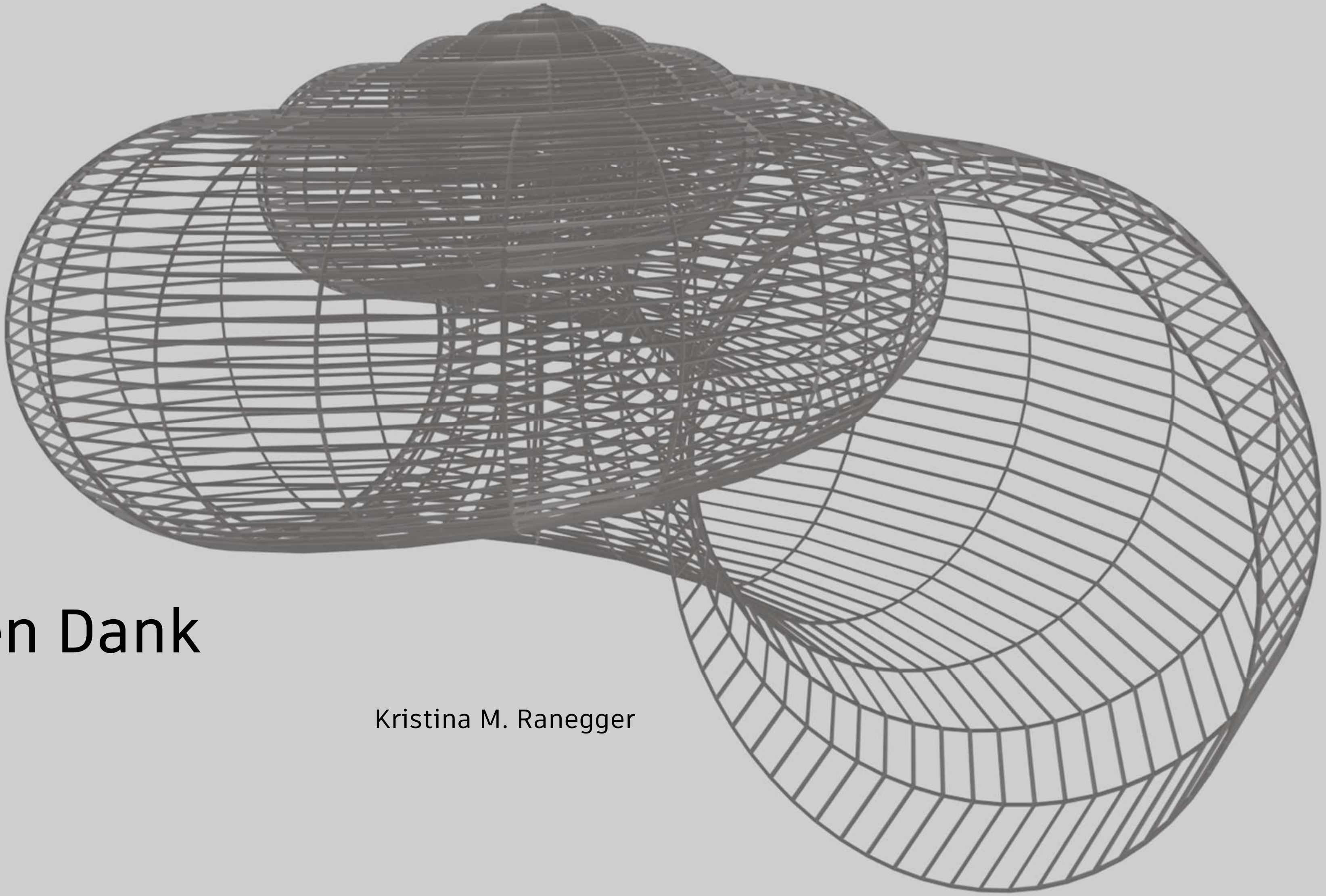
 

gerade geschlossene Strahlspiralflächen





schiefe offene Strahlspiralflächen



Φ-len Dank

Kristina M. Ranegger