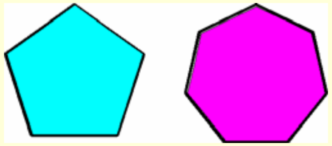
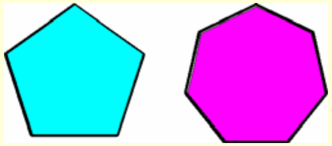


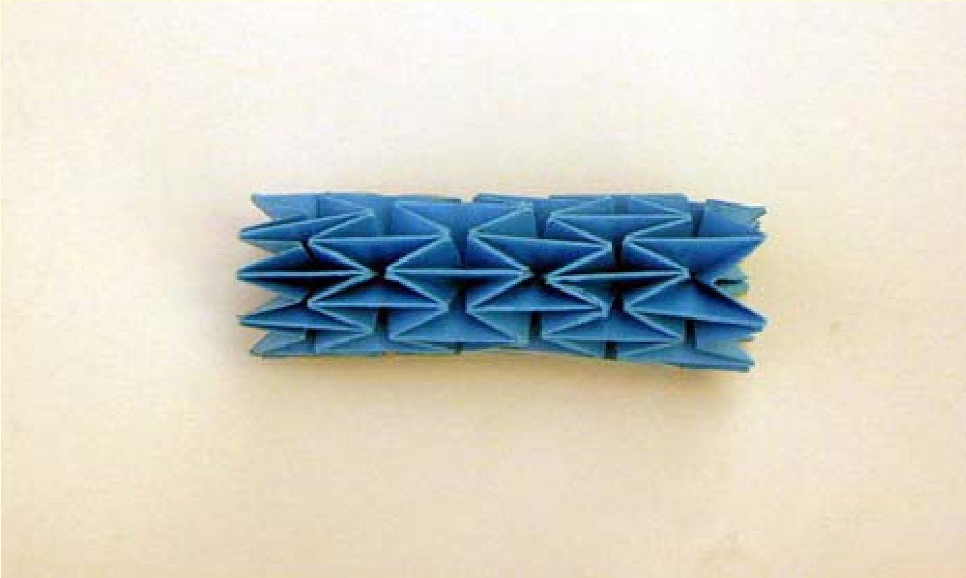
Origami und Geometrie – Papier kann mehr als man denkt

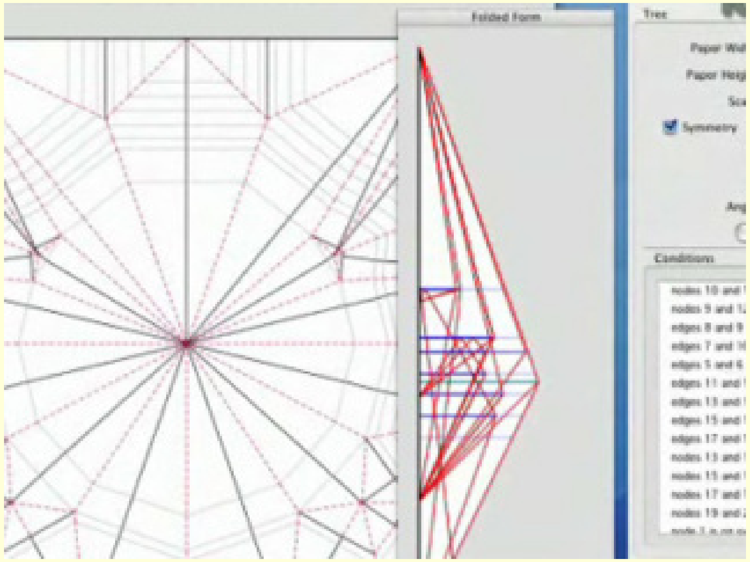
Robert Geretschläger

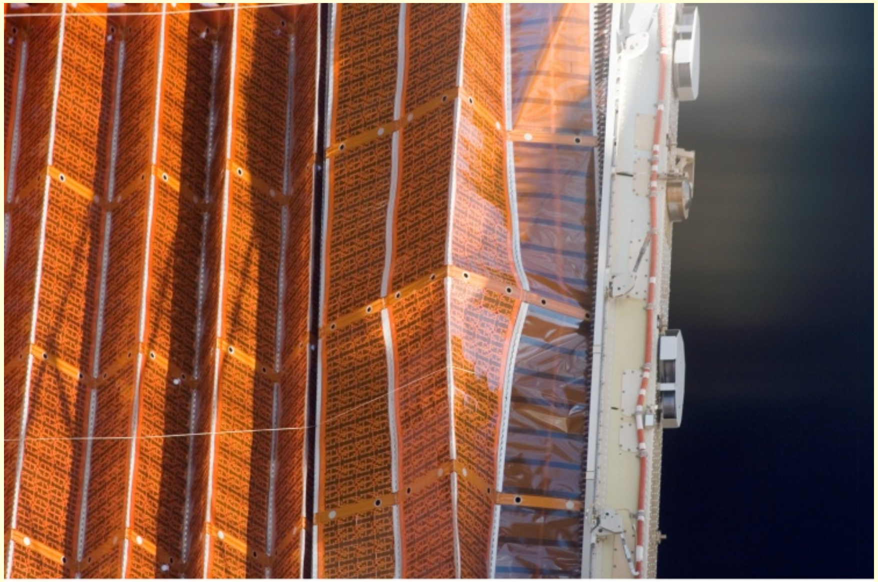
BRG Kepler, Graz

Strobl, 7.11.2012









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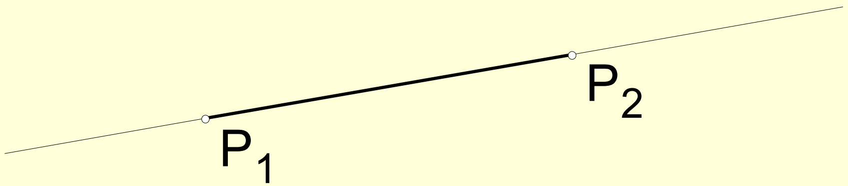
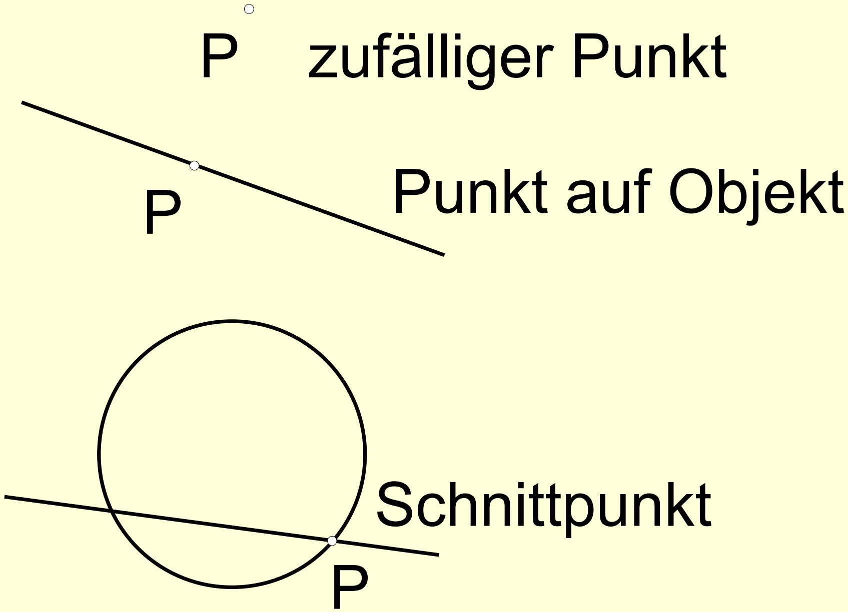
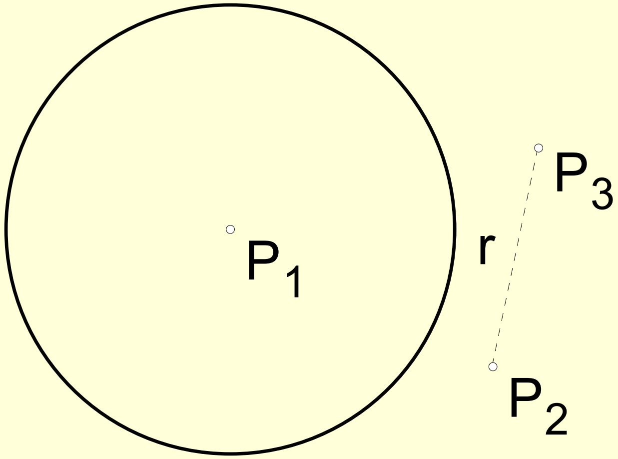
Zirkel und Lineal:

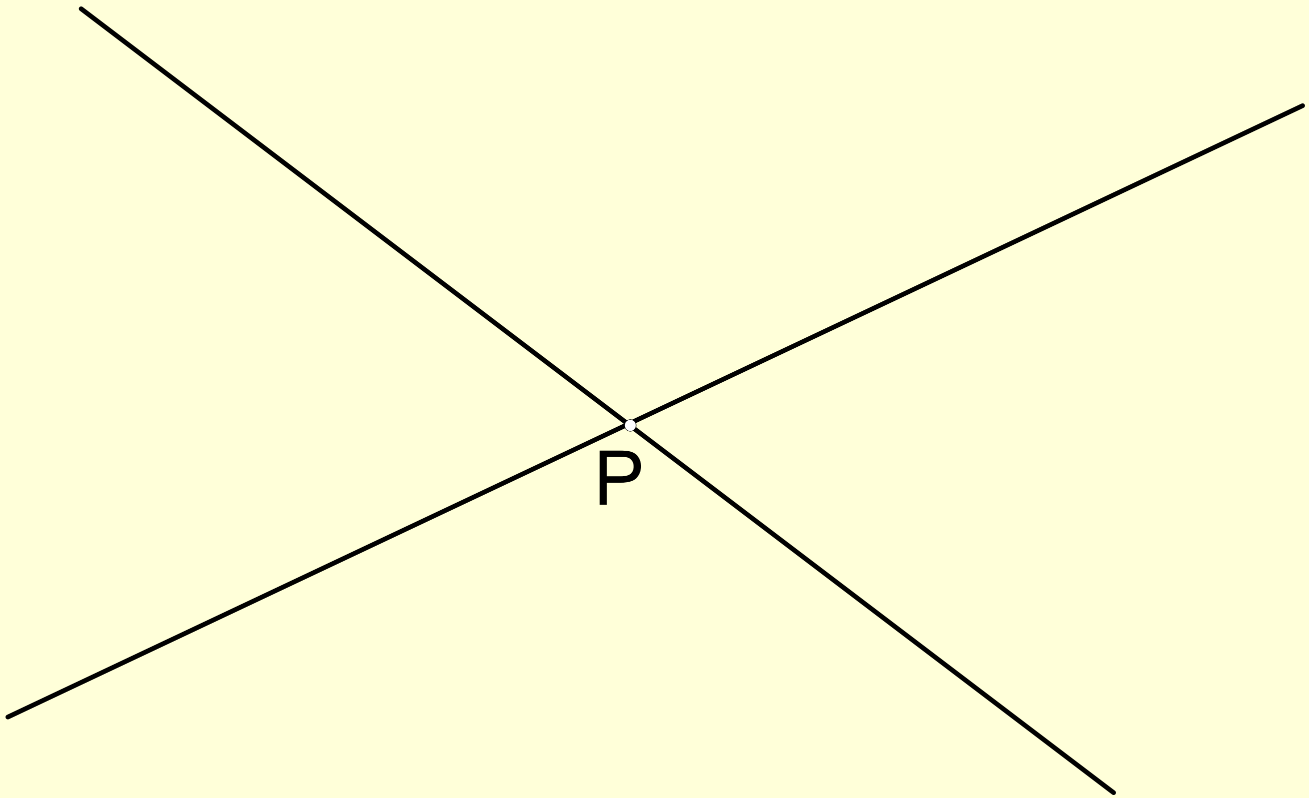






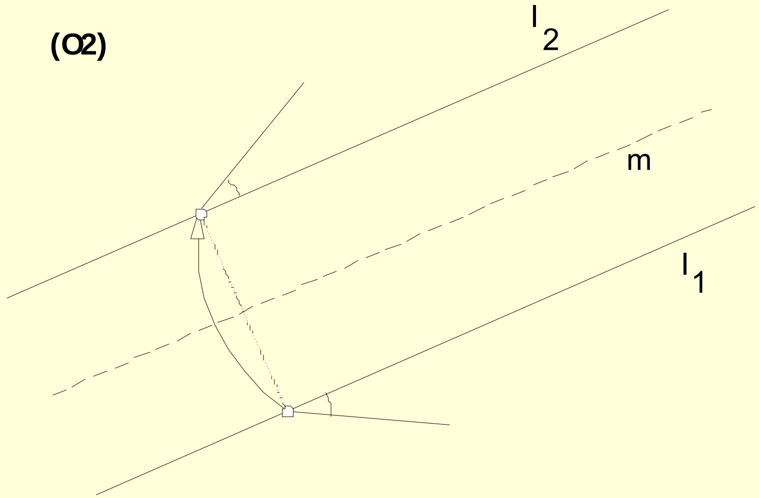
Papierfalten:

(O1)



Konstruktionsaxiome

(O2)



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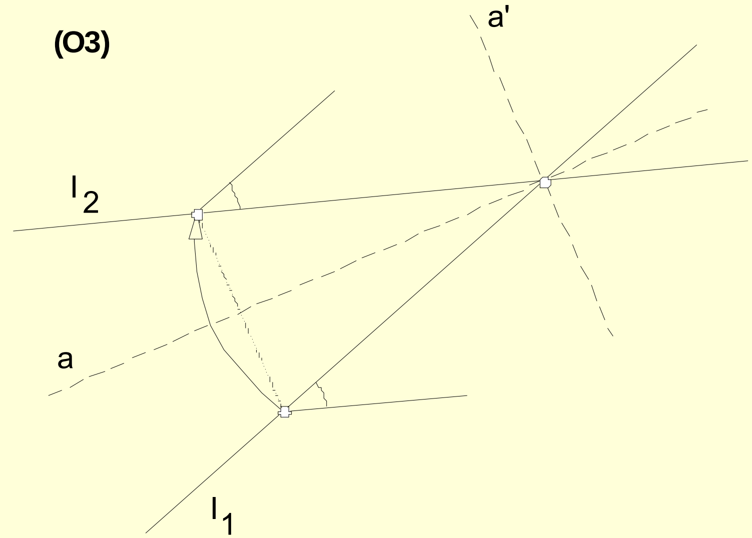
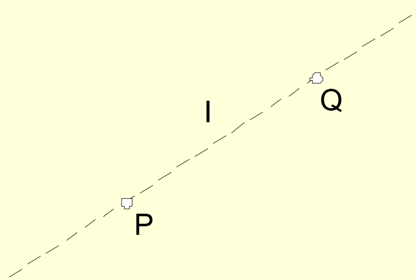
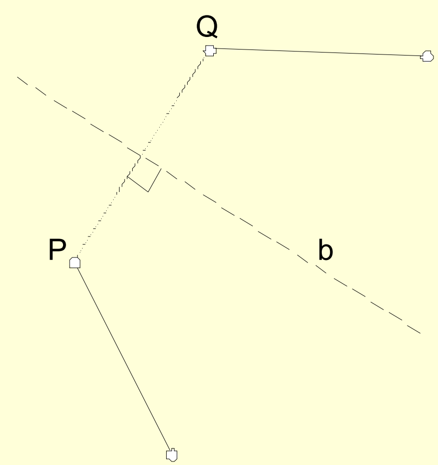


(O4)



(O5)



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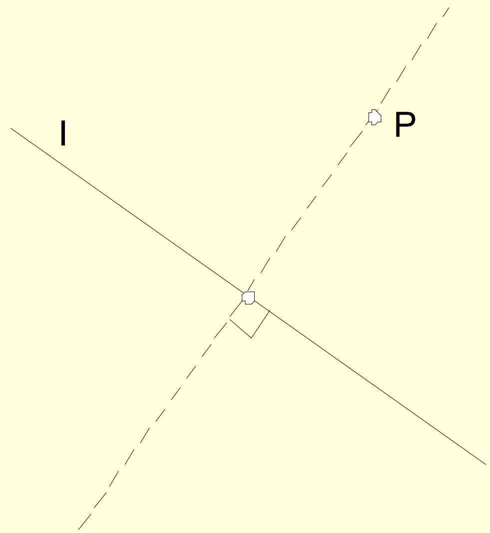
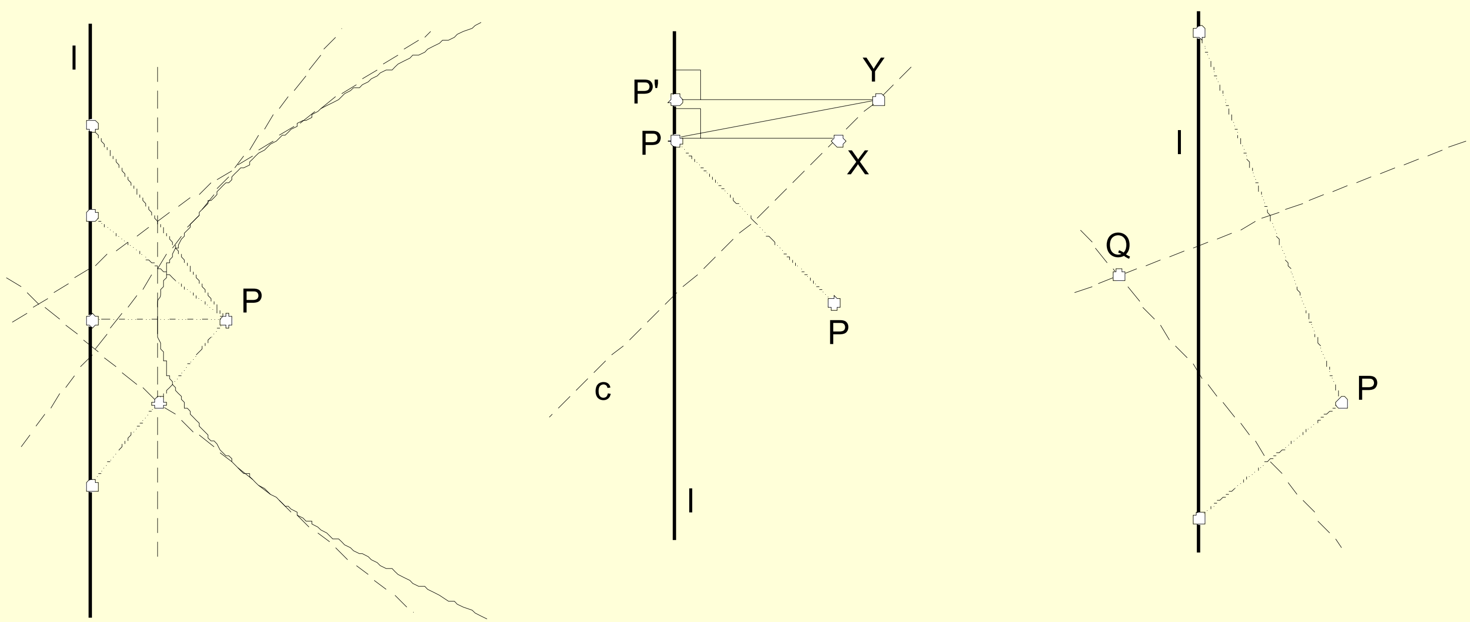
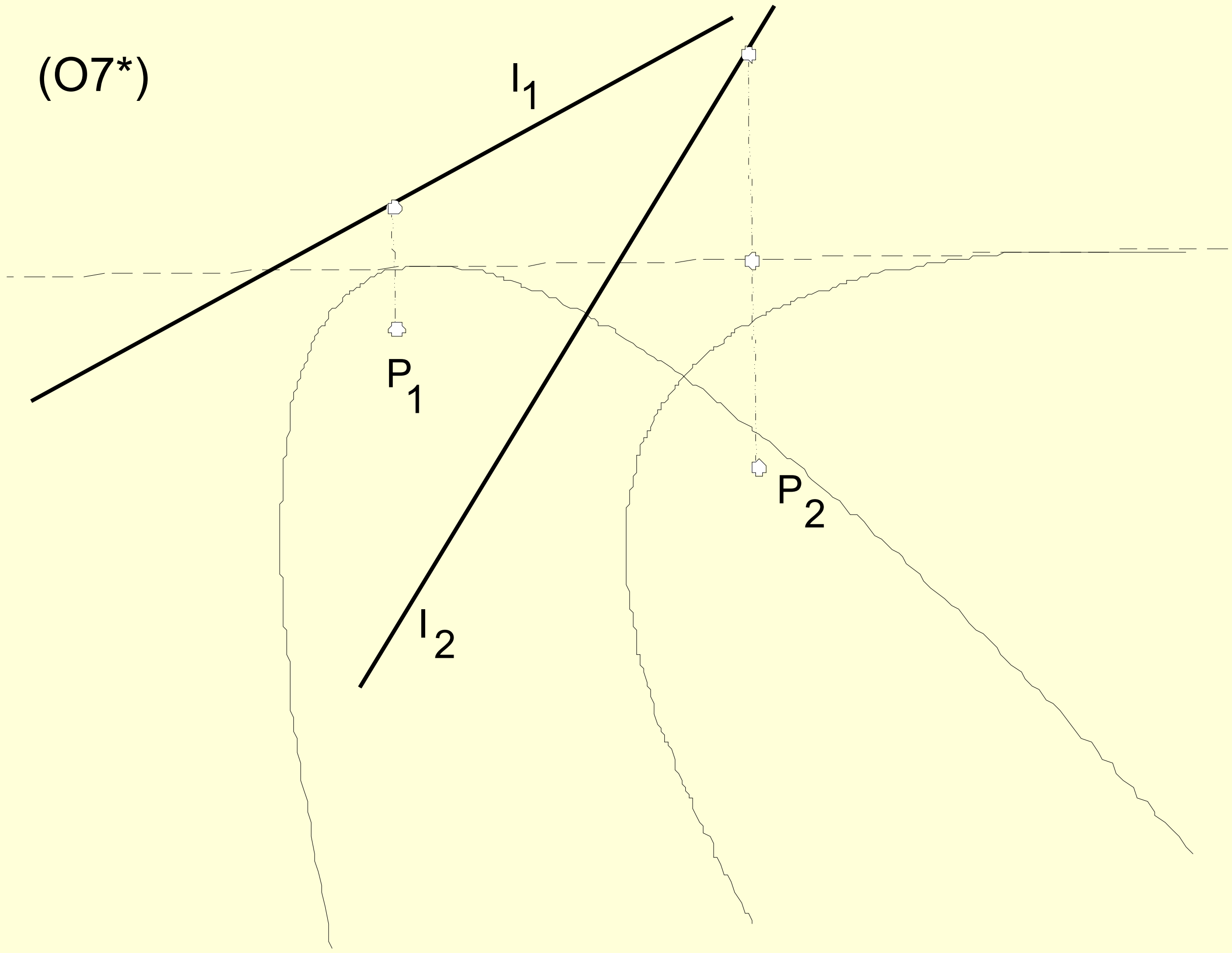
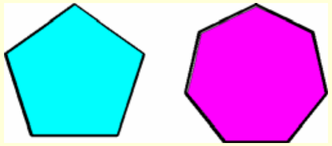


(O7)

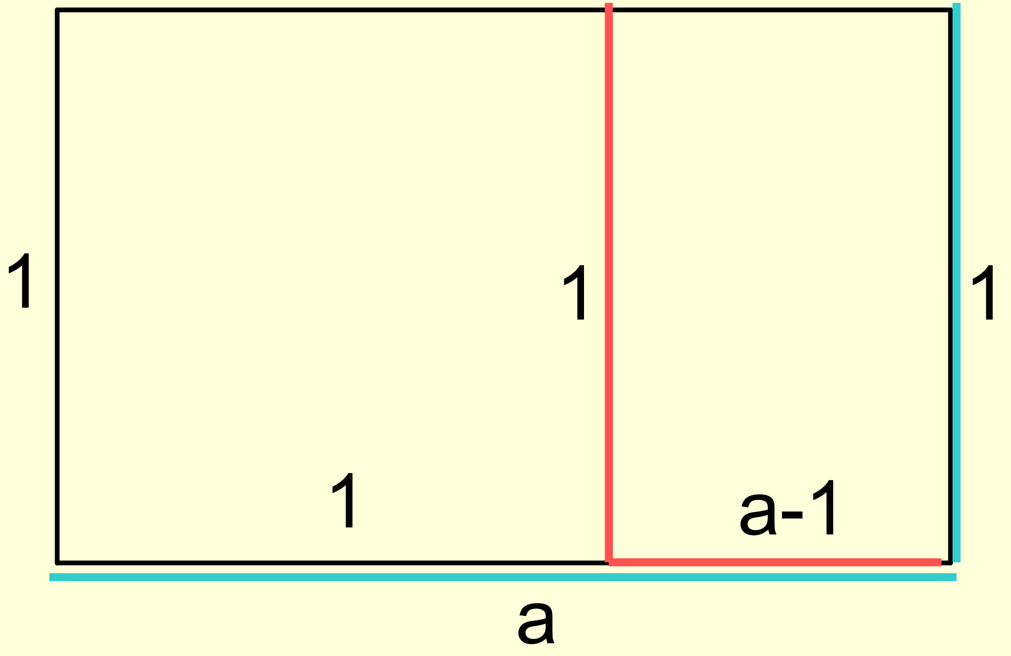


(O7*)





Der Goldene Schnitt



$$a : 1 = 1 : (a-1)$$

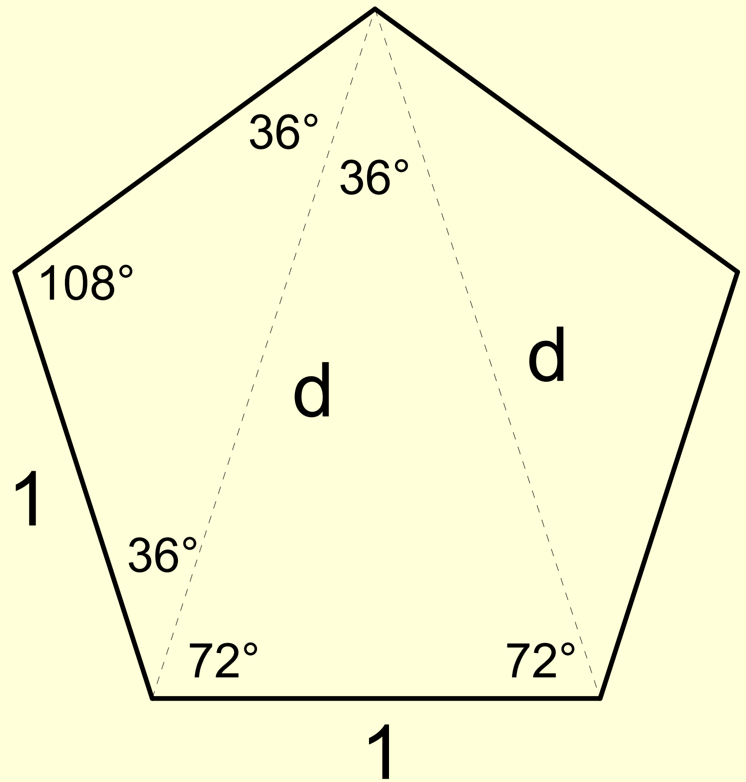
$$\Leftrightarrow \quad a^2 - a = 1$$

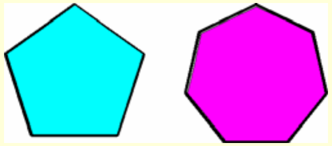
$$\Leftrightarrow \quad a^2 - a - 1 = 0$$

$$\Leftrightarrow \quad a = \frac{\sqrt{5}+1}{2}$$

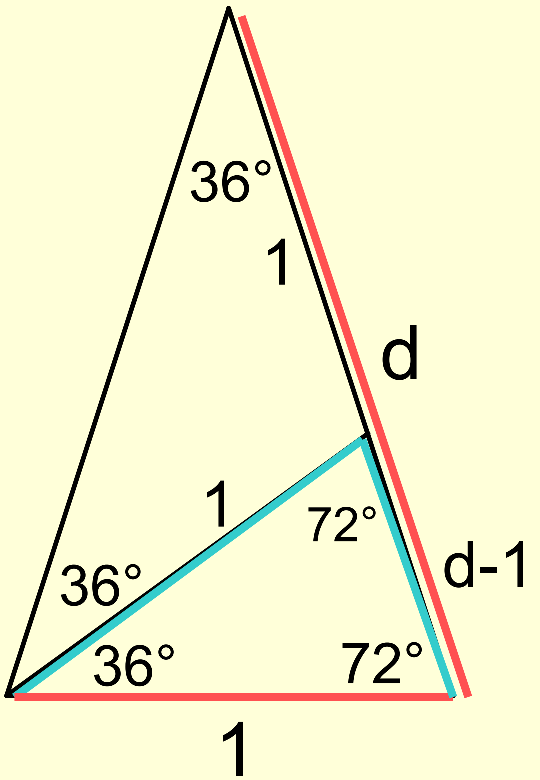
$$\phi = a : 1$$

Winkel im regelmäßigen 5-eck



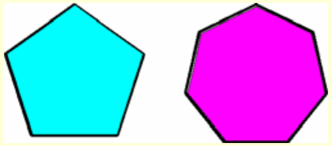


Das Goldene Dreieck

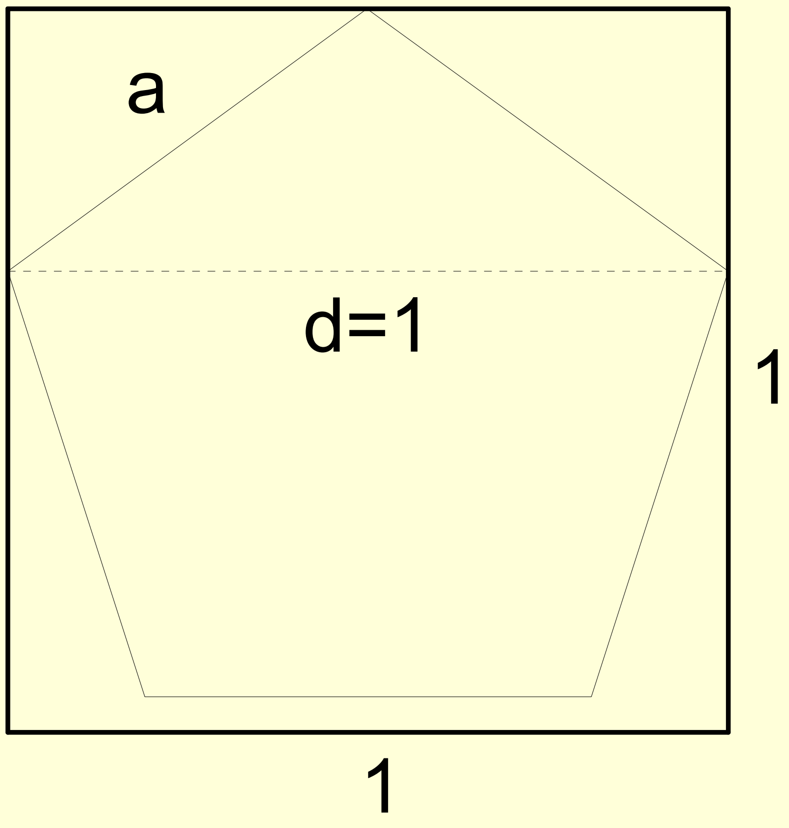


$$d : 1 = 1 : (d-1)$$

$$\Leftrightarrow \quad d = \frac{\sqrt{5}+1}{2} = \phi$$

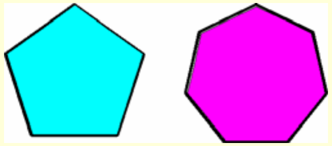


Platzierung des Pentagons auf dem Papier

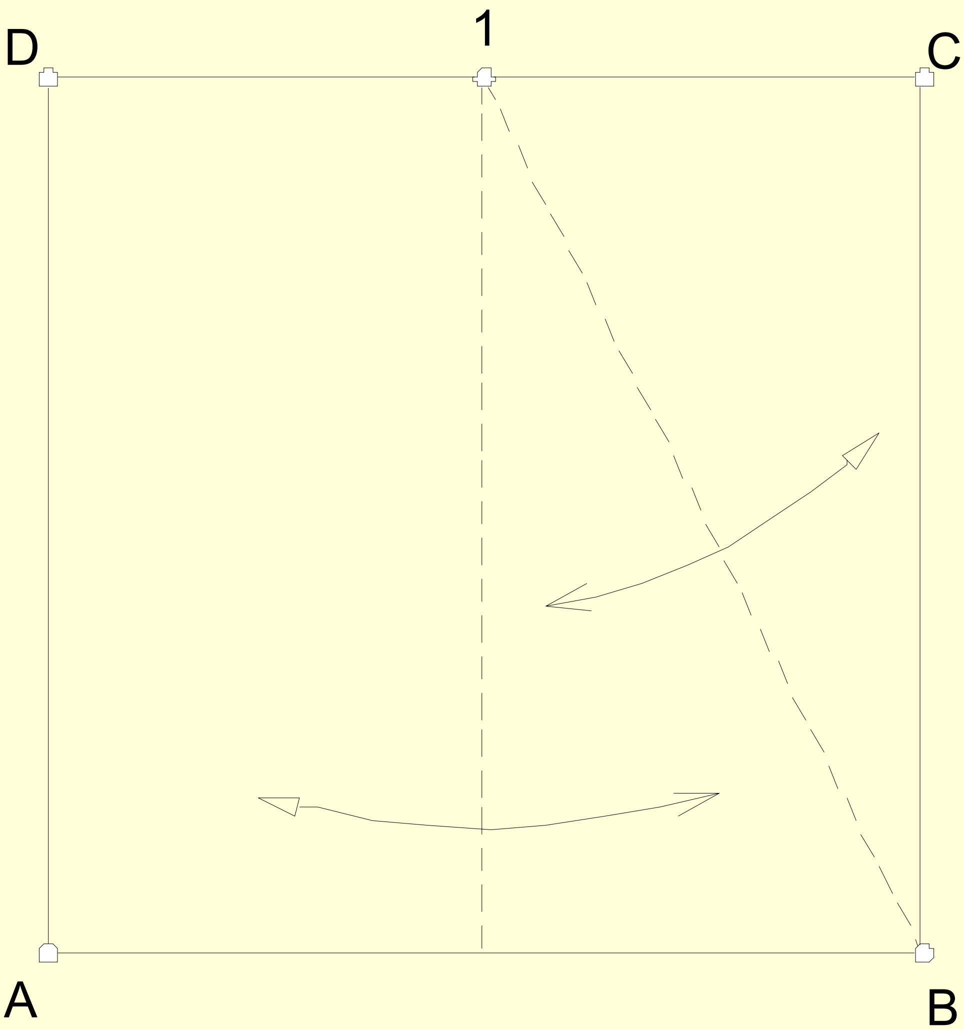


$$\frac{a}{d}=\frac{1}{\phi}=\frac{2}{\sqrt{5}+1}=\frac{2\cdot\left(\sqrt{5}-1\right)}{\left(\sqrt{5}+1\right)\cdot\left(\sqrt{5}-1\right)}=\frac{2\cdot\left(\sqrt{5}-1\right)}{4}=\frac{\sqrt{5}-1}{2}$$

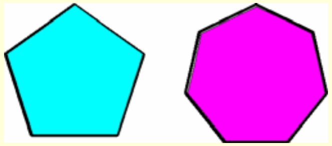
5-eck



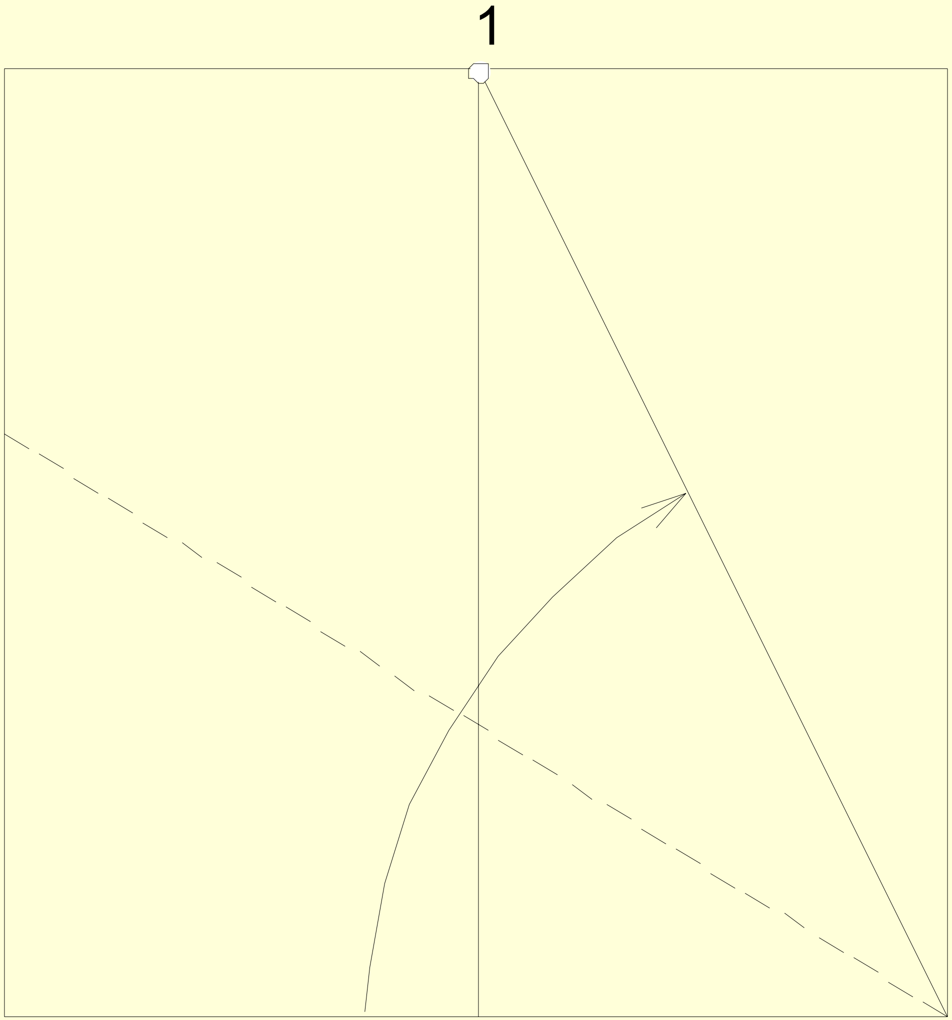
Schritt 1



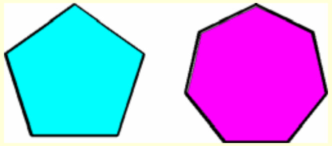
5-eck



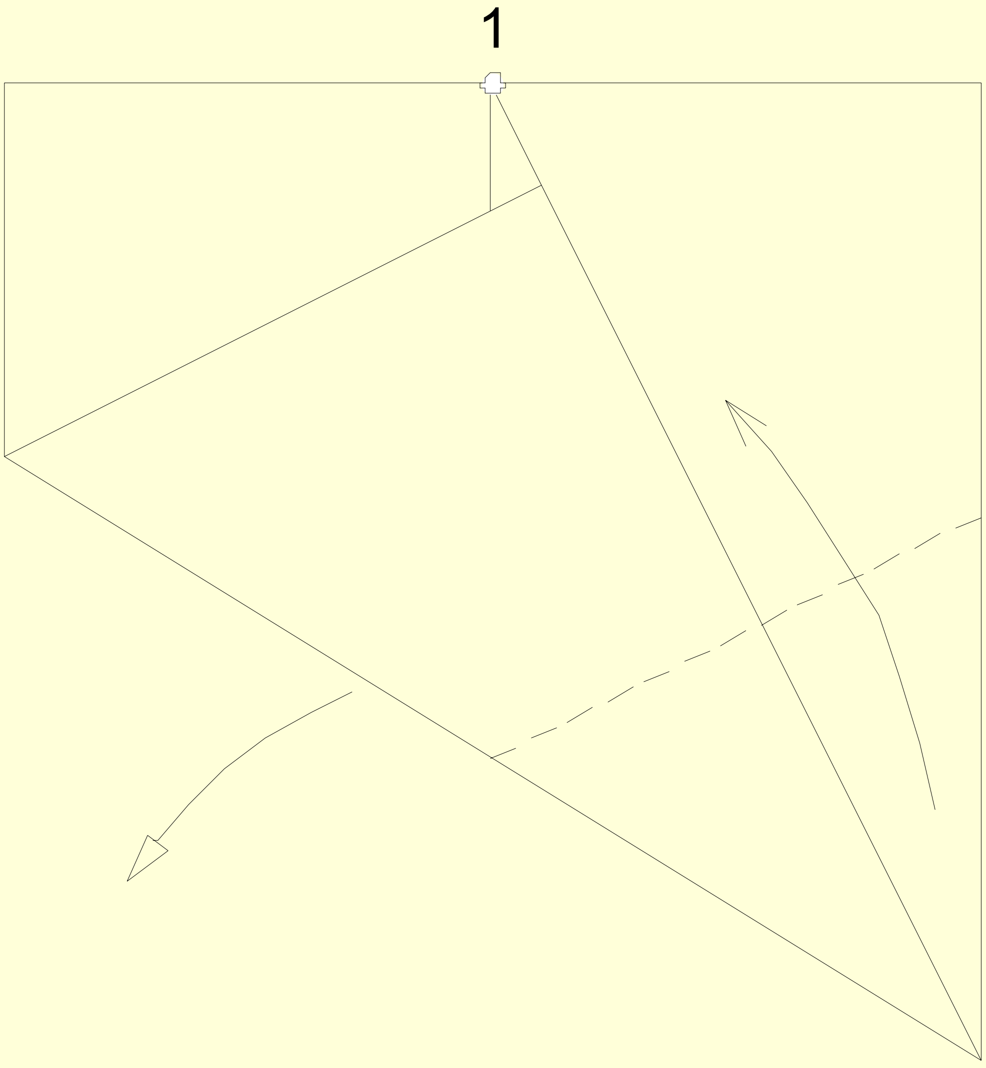
Schritt 2



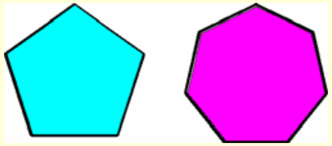
5-eck



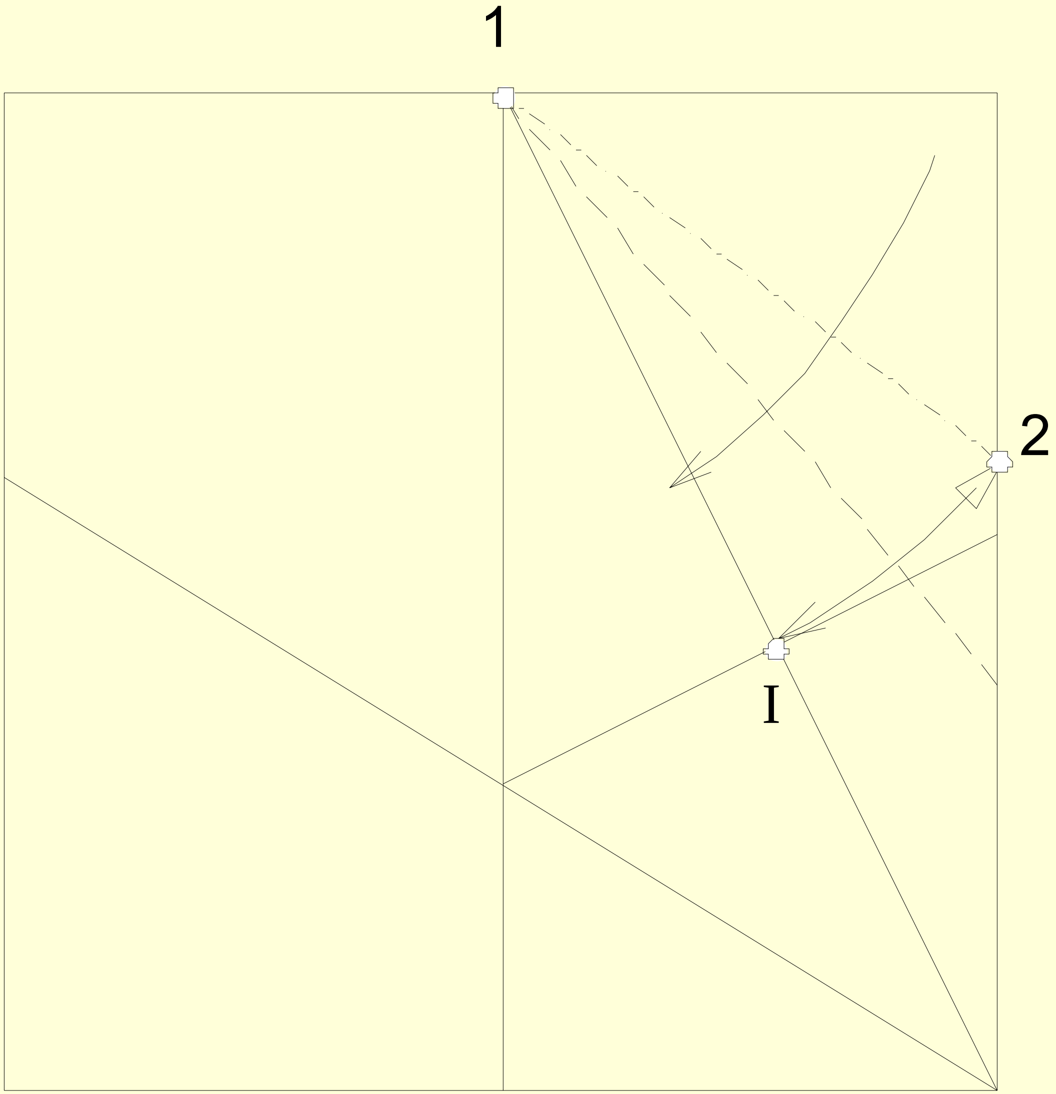
Schritt 3



5-eck



Schritt 4



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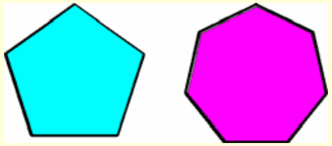
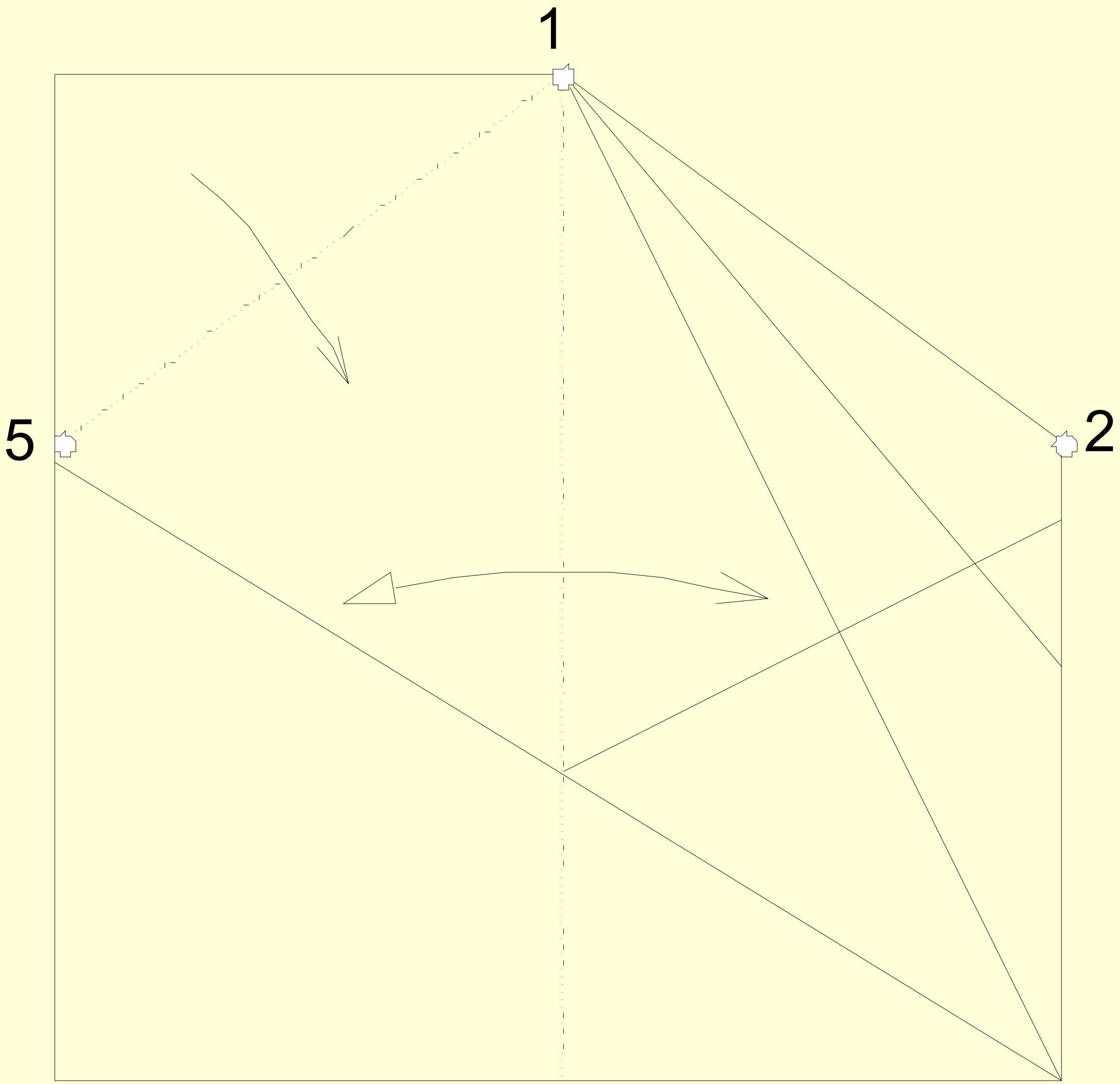


Schritt 5



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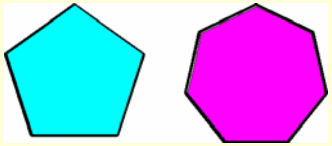
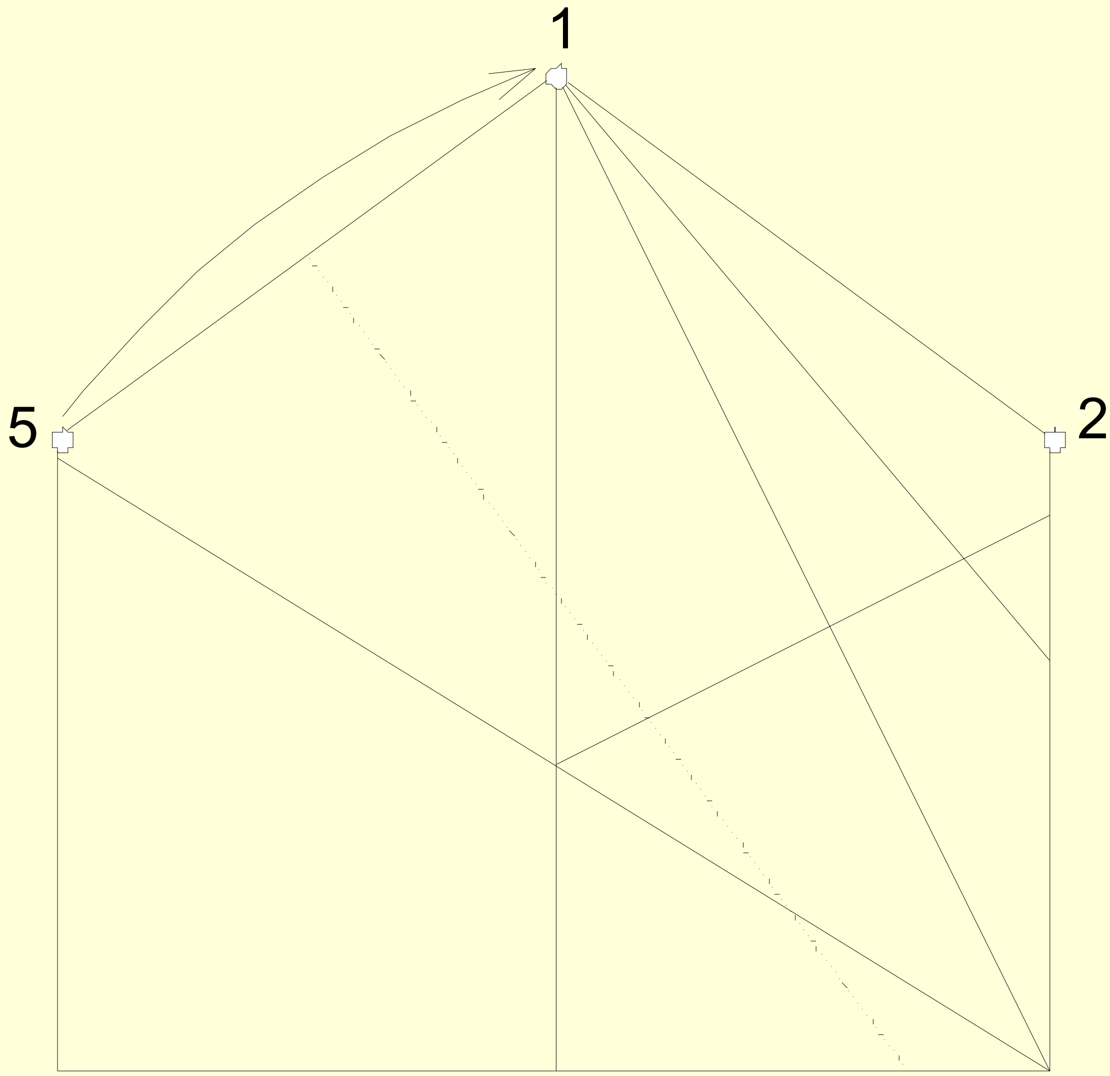


Schritt 6



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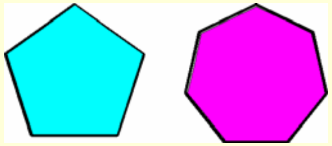
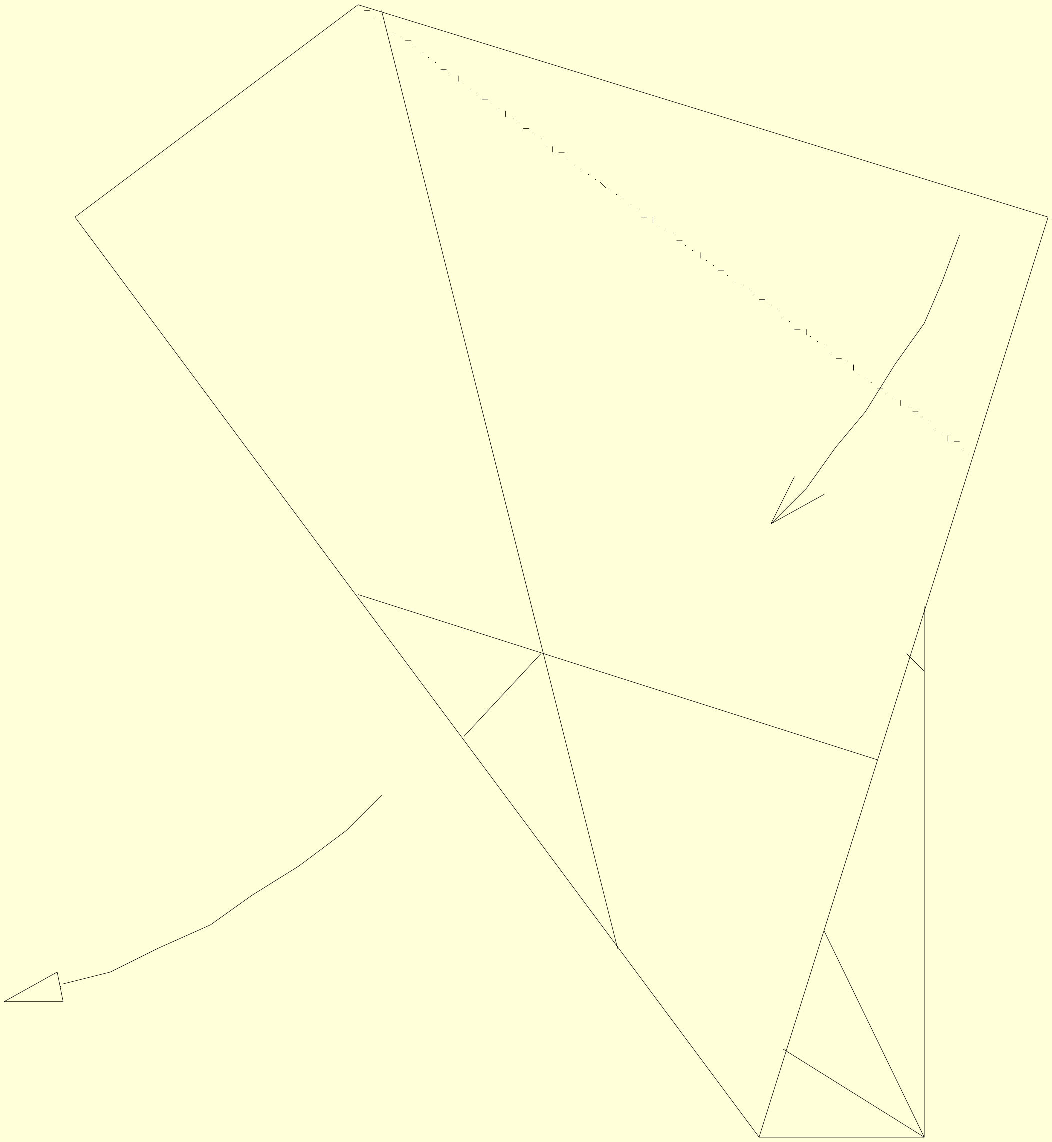


Schritt 7



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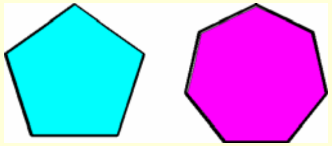
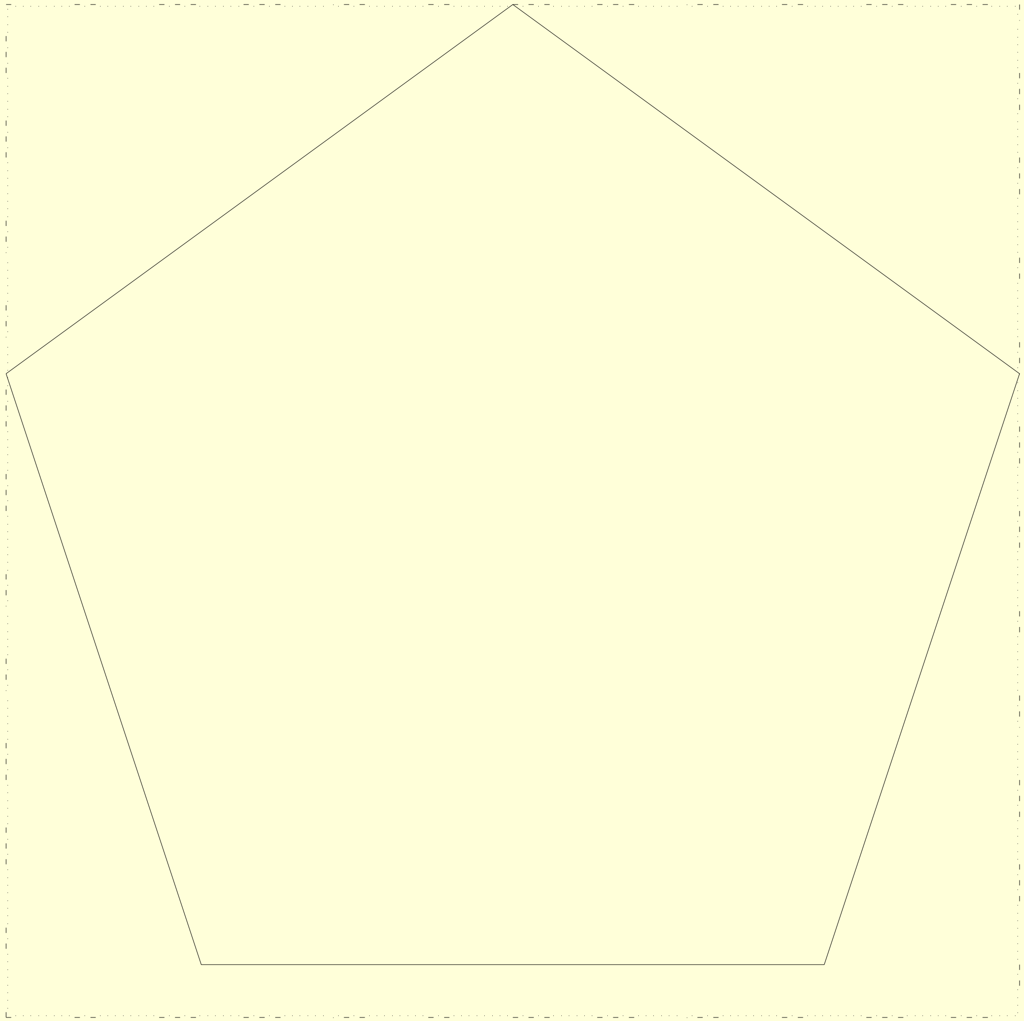
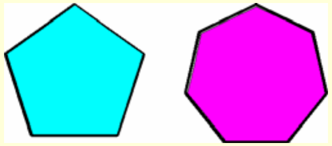


Schritt 8





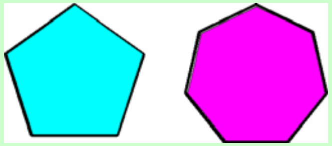
weitere Herausforderungen für fortgeschrittene Pentagonisten:

+++ Kann ein regelmäßiges 5-eck mit Seitenlänge *a* größer als $1/\phi$ im Inneren eines Einheitsquadrats platziert werden?

+++ Bestimme eine Faltsequenz für ein größeres regelmäßiges Fünfeck.

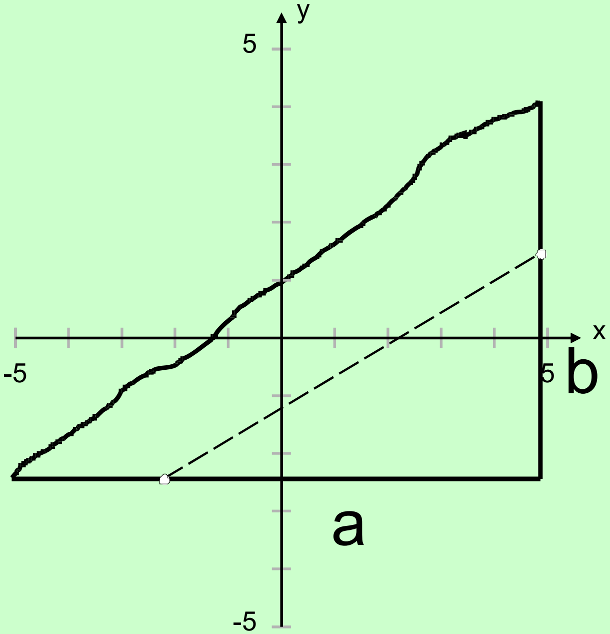
+++ Bestimme den größtmöglichen Wert von *a*. Beweise, dass dieser Wert tatsächlich maximal ist.

Gleichung 1. Grades



lineare Gleichung ax = b

Lösung: $x = \dfrac{b}{a}$



Steigung der Faltkante ist $\dfrac{b}{a}$

Gleichung 2. Grades

Quadratische Gleichung $x^2+px+q = 0$

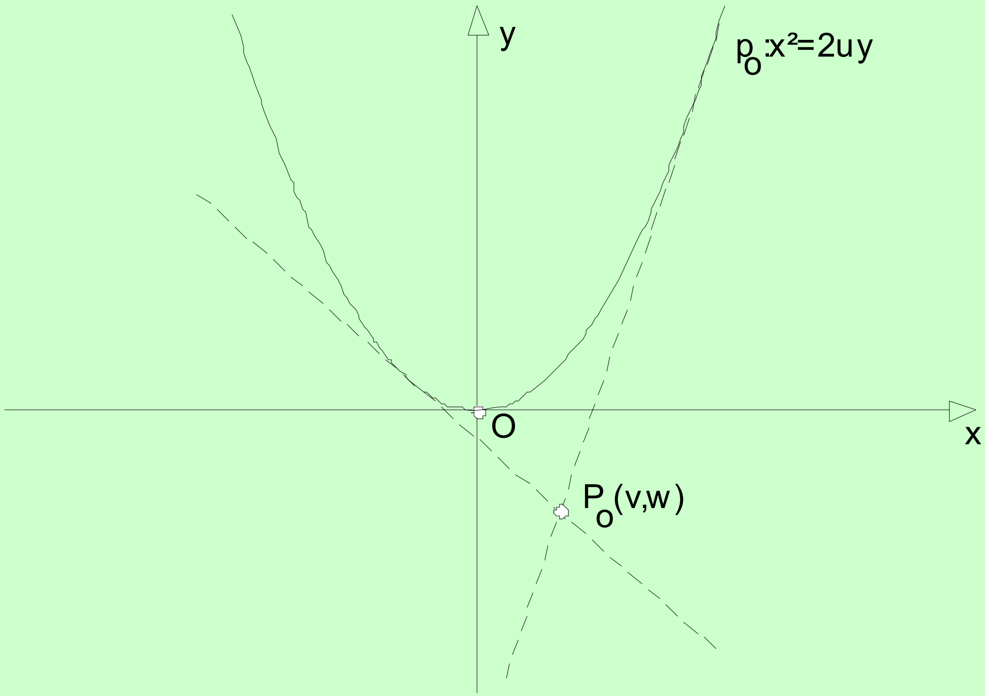
Parabel:
$x^2 = 2u \cdot y$

Tangente:
$y = s \cdot (x - v) + w$

$x^2 - 2usx + 2uvs - 2uw = 0$

$u^2s^2 - 2uvs + 2uw = 0$

$$s^2 - \frac{2v}{u} \cdot s + \frac{2w}{u} = 0$$



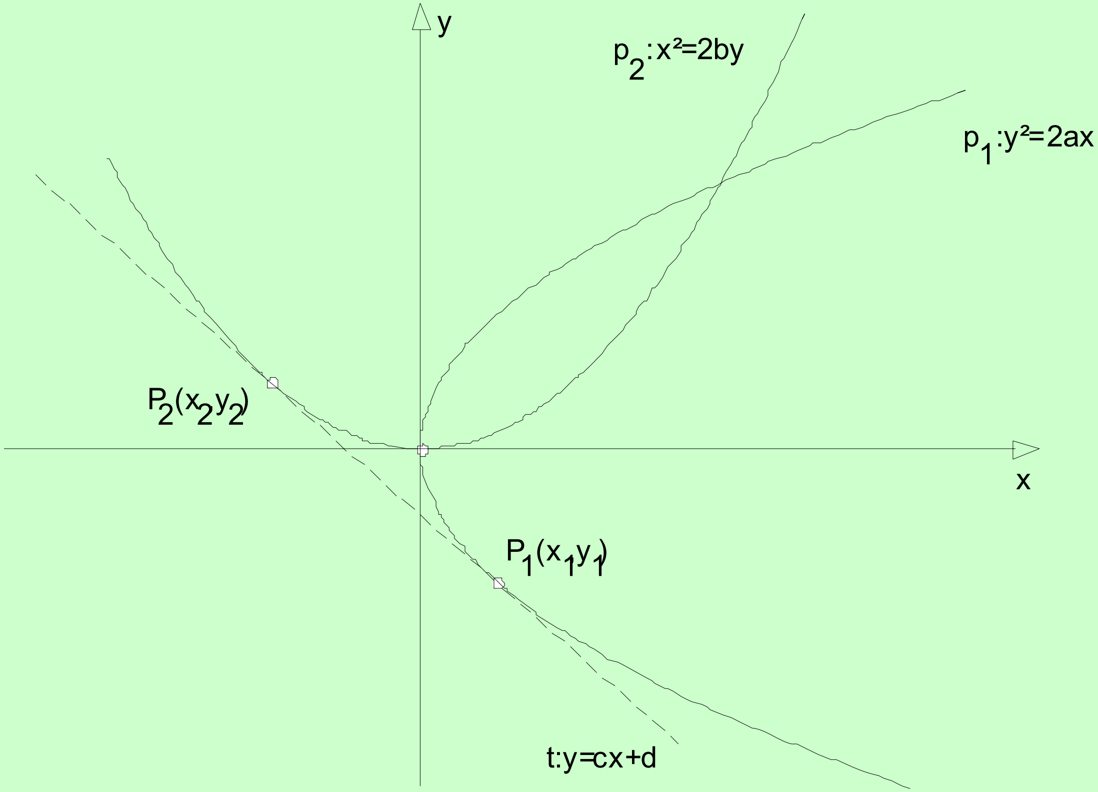
$u = 2$, $v = -p$, $w = q$

Parabel: $x^2 = 4y$

(Brennpunkt F(0/1),
Leitlinie $y = -1$)

$P_0(-p,q)$

Gleichung 3. Grades



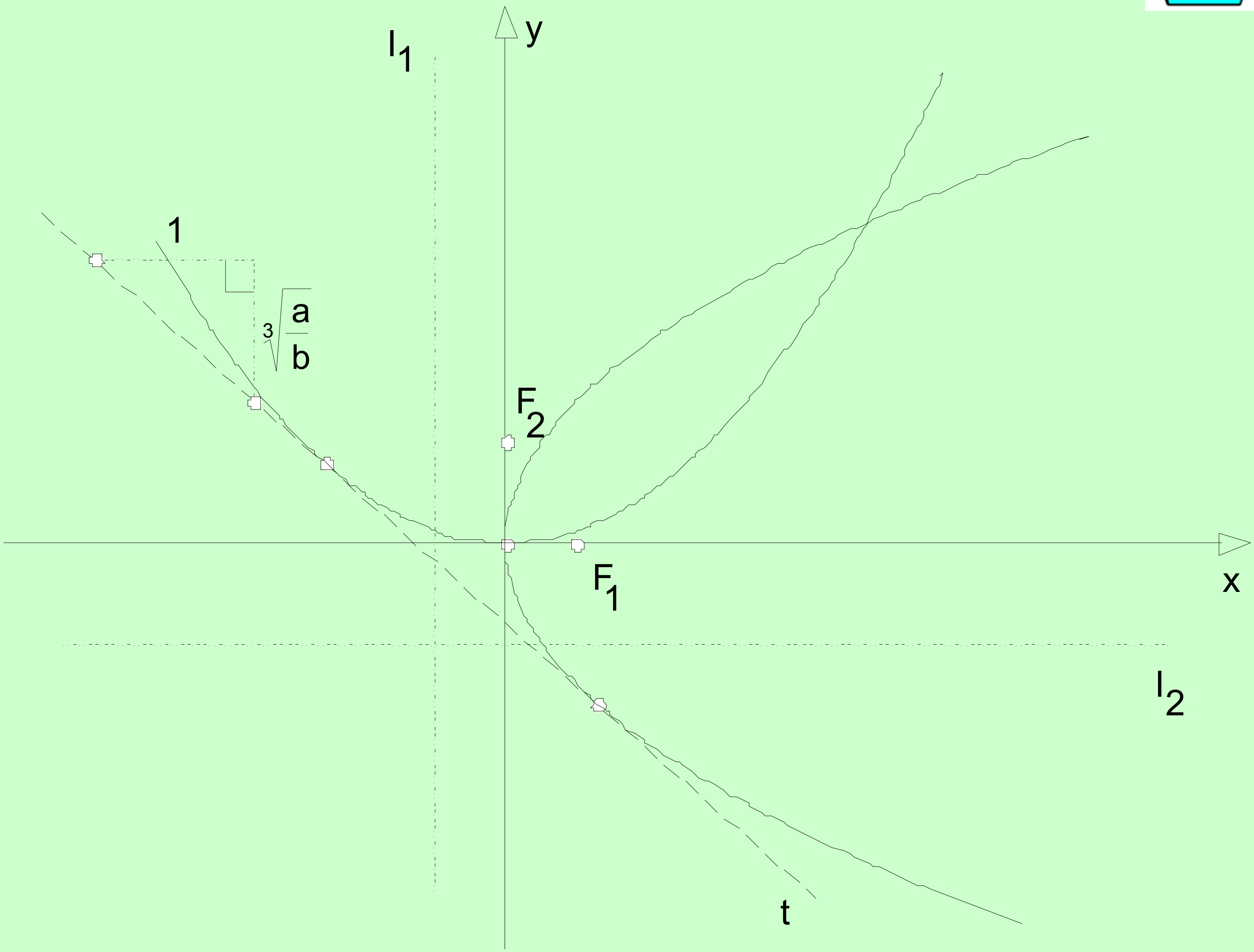
$$t: y = cx + d$$

$$p_1: yy_1 = ax + ax_1$$

$$p_2: xx_2 = by + by_2$$

$$c = -\sqrt[3]{\frac{a}{b}}$$

Gleichung 3. Grades



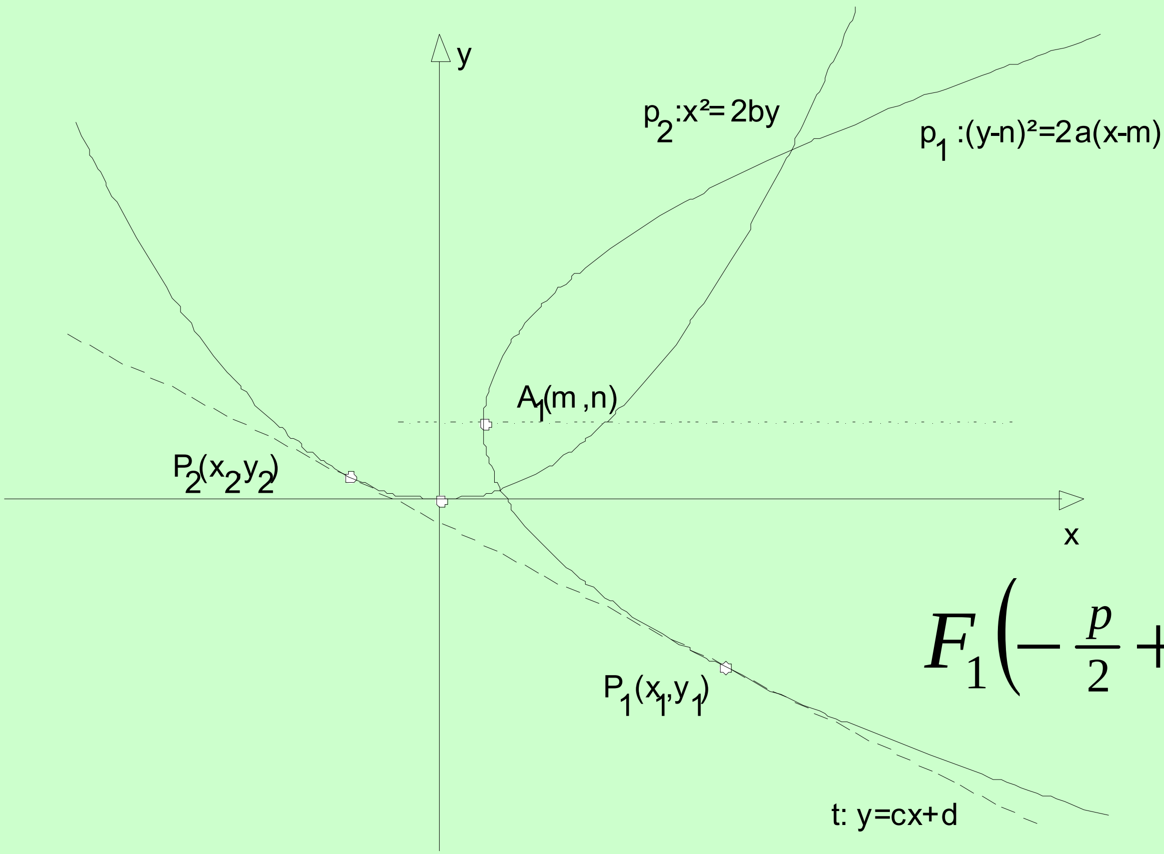
Gleichung 3. Grades

t: $y = cx + d$

p_1: $(y-n)(y_1-n) = a(x-m) + a(x_1-m)$

p_2: $xx_2 = by + by_2$

$$c^3 - \frac{2m}{b} \cdot c^2 + \frac{2n}{b} \cdot c + \frac{a}{b} = 0$$



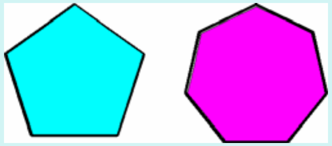
$x^3 + px^2 + qx + r = 0$

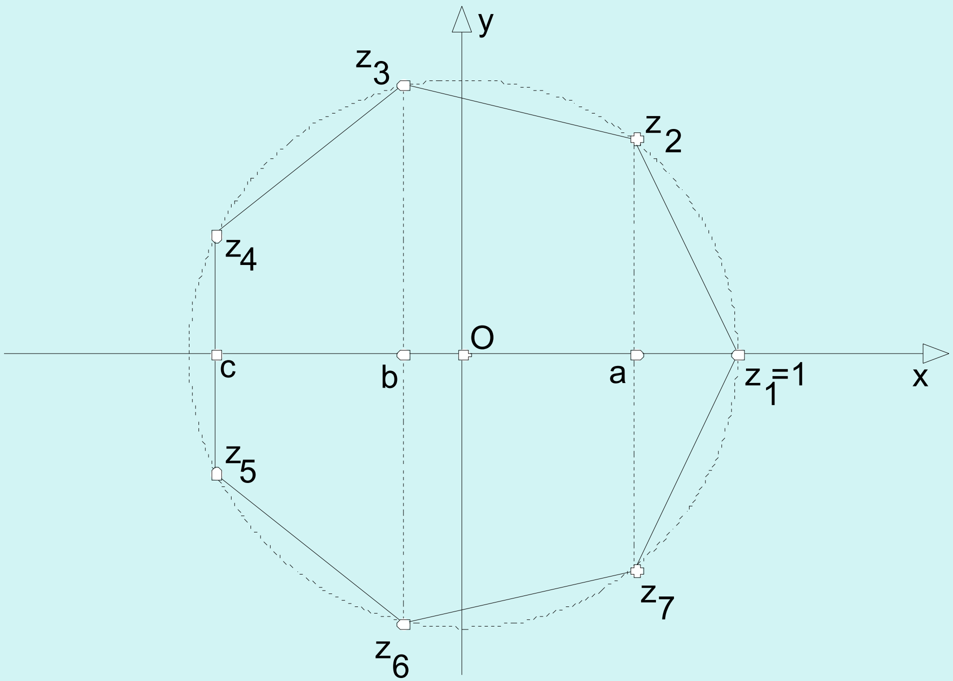
$p = -2m$, $q = 2n$,

$r = a$, $b = 1$

$$F_1\left(-\frac{p}{2} + \frac{r}{2}, \frac{q}{2}\right); \; l: x = -\frac{p}{2} - \frac{r}{2}$$

7-eck





$$z^7 - 1 = 0$$

$$\frac{z^7-1}{z-1} = z^6 + z^5 + z^4 + z^3 + z^2 + z^1 + 1 = 0.$$

$$z^3 + z^2 + z + 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} = 0$$

$$\zeta^3 \quad = \quad \left(z + \frac{1}{z}\right)^3$$

$$\zeta^3 + \zeta^2 - 2\zeta - 1 = 0$$

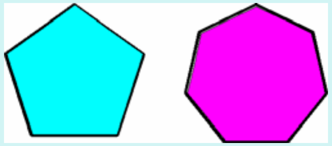
$F_1(-1,-1)$

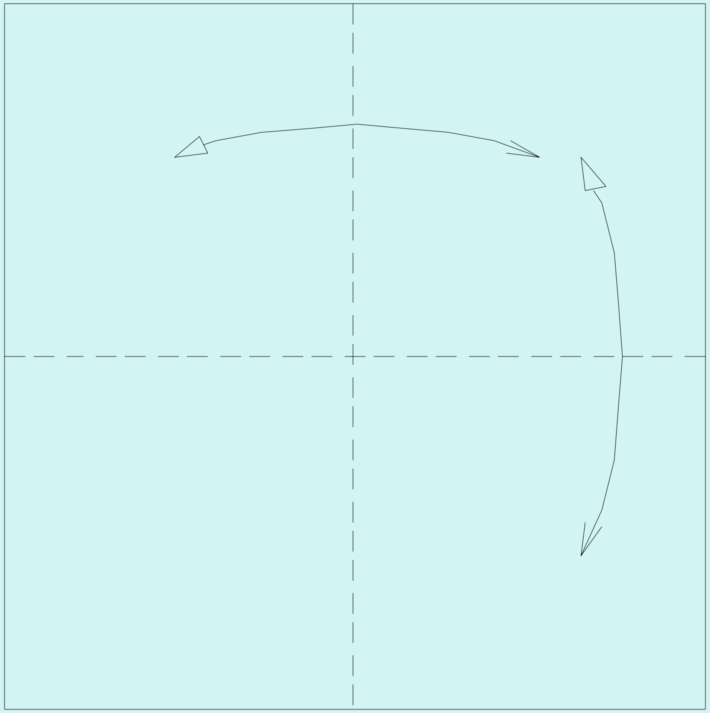
$l_1 : x = 0$

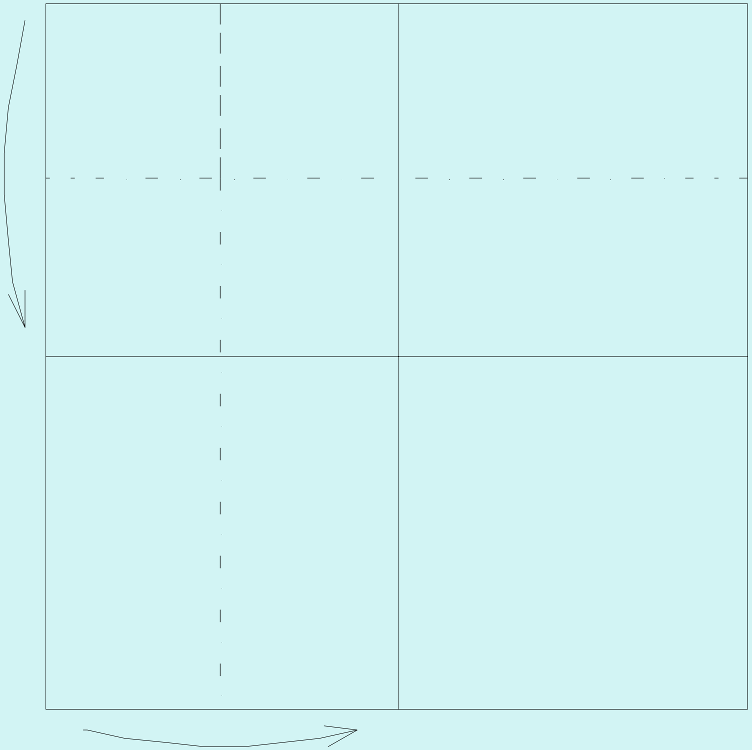
$F_2\left(0, \frac{1}{2}\right)$

$l_2 : y = -\frac{1}{2}$

7-eck







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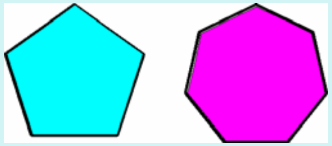

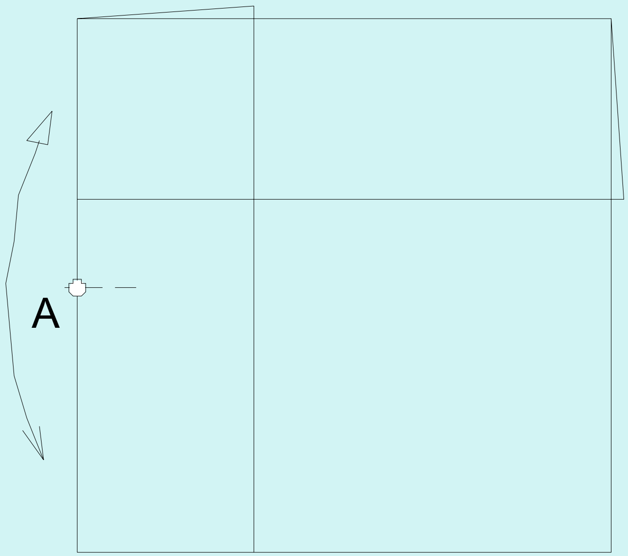

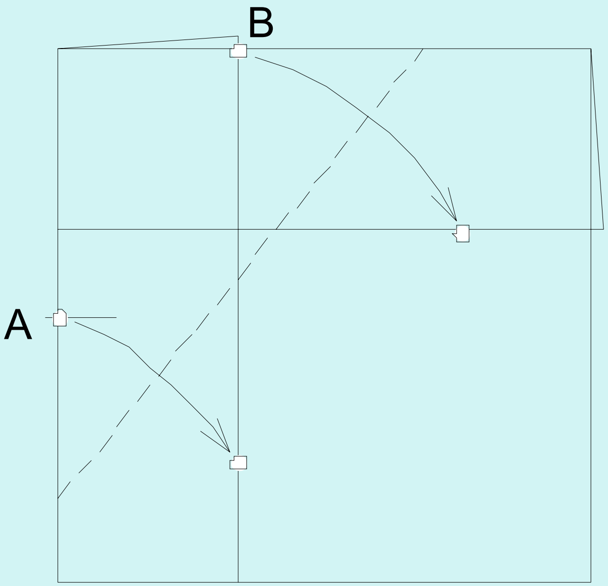

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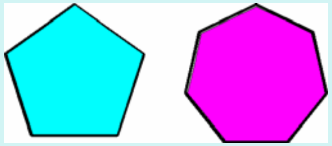

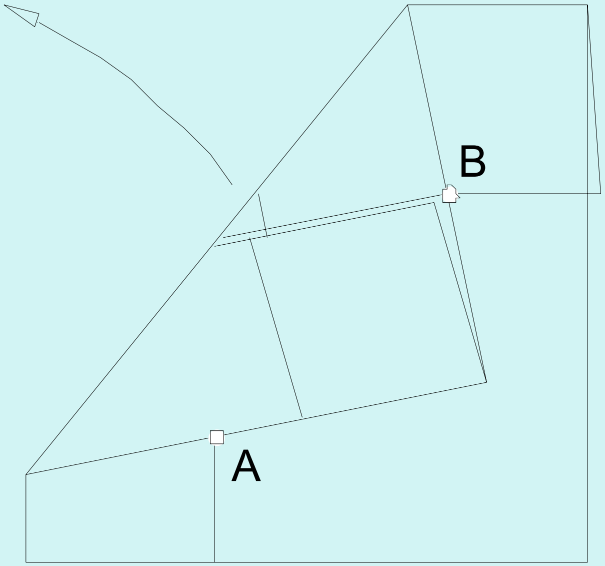

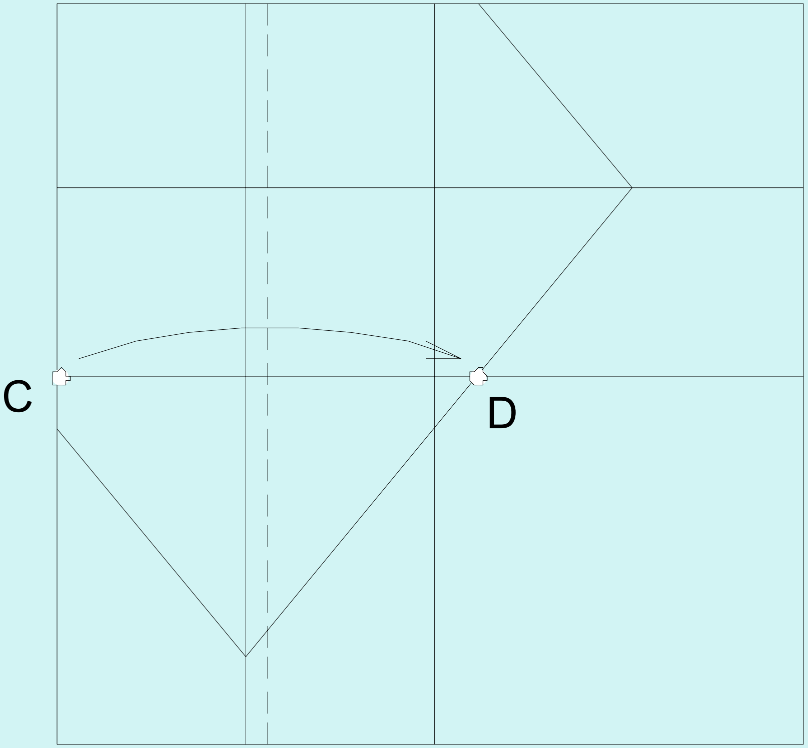

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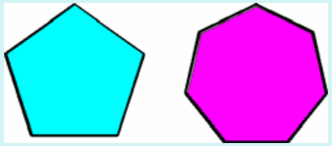

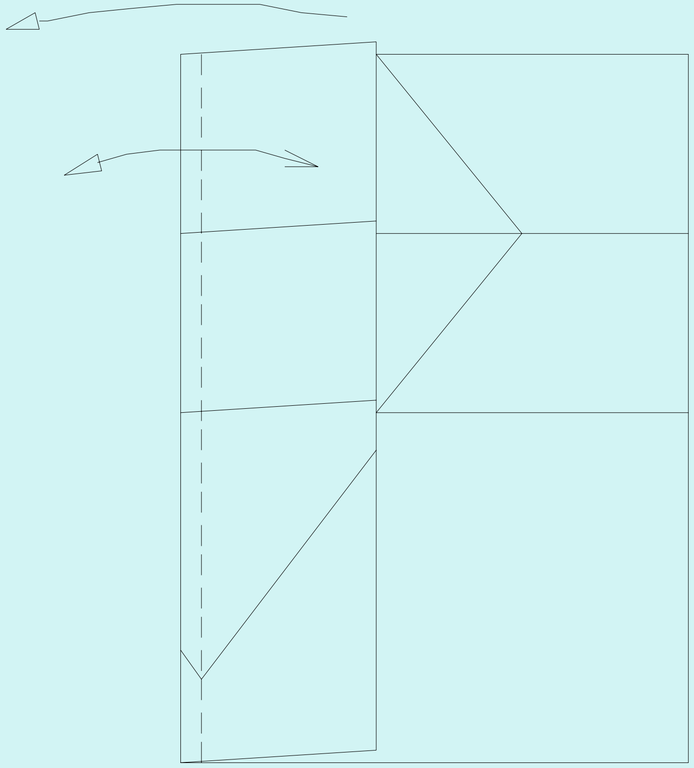

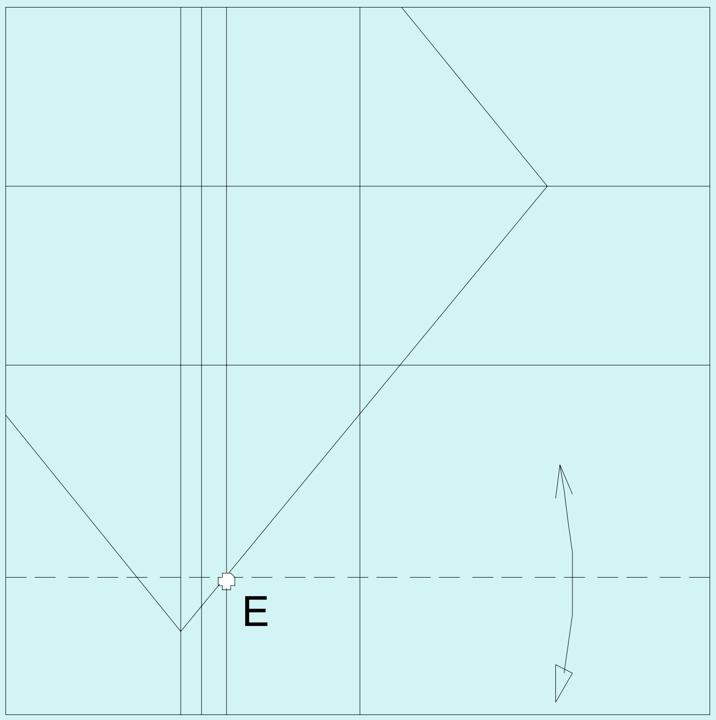

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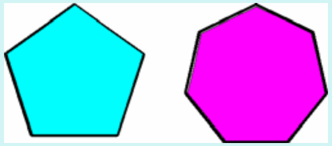

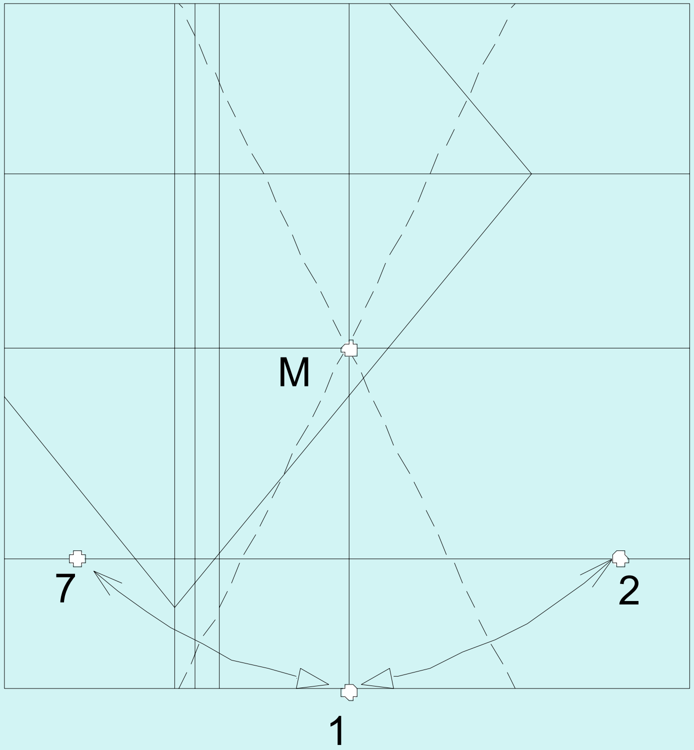

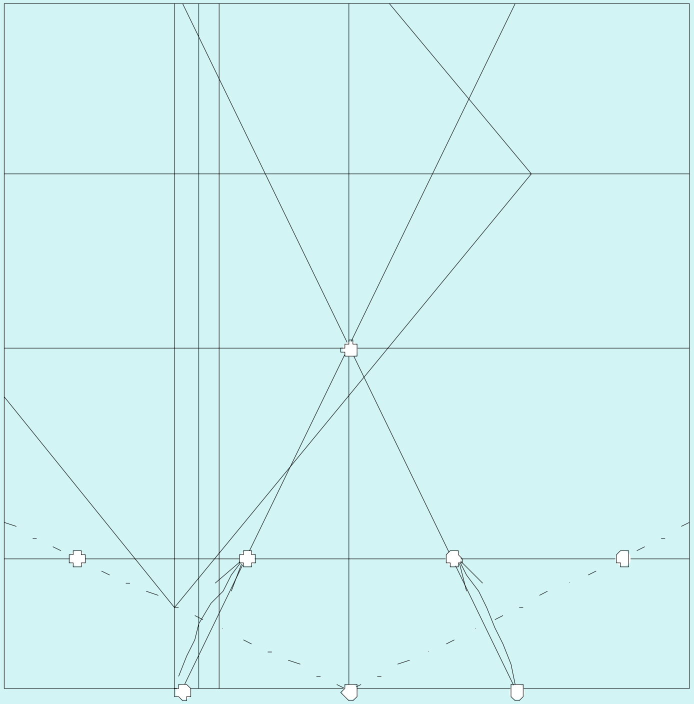

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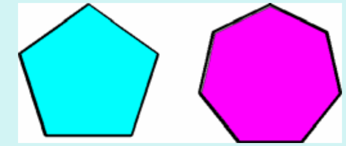

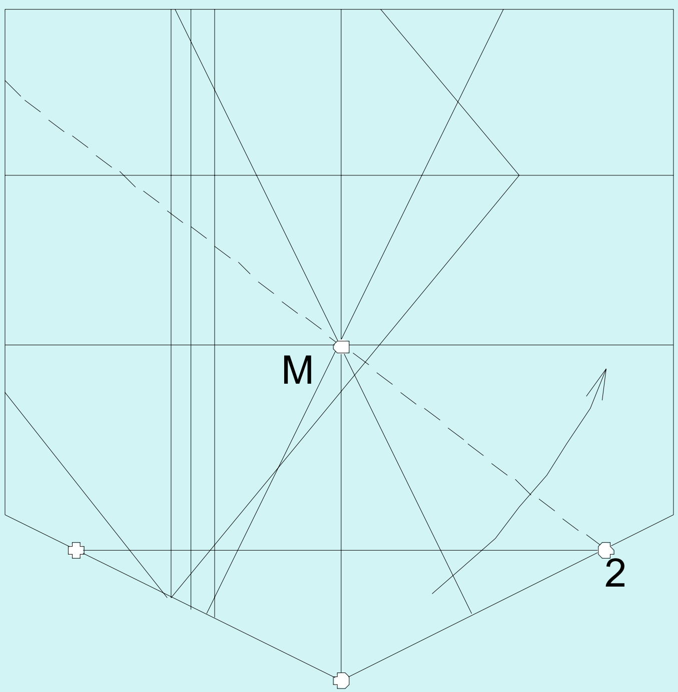

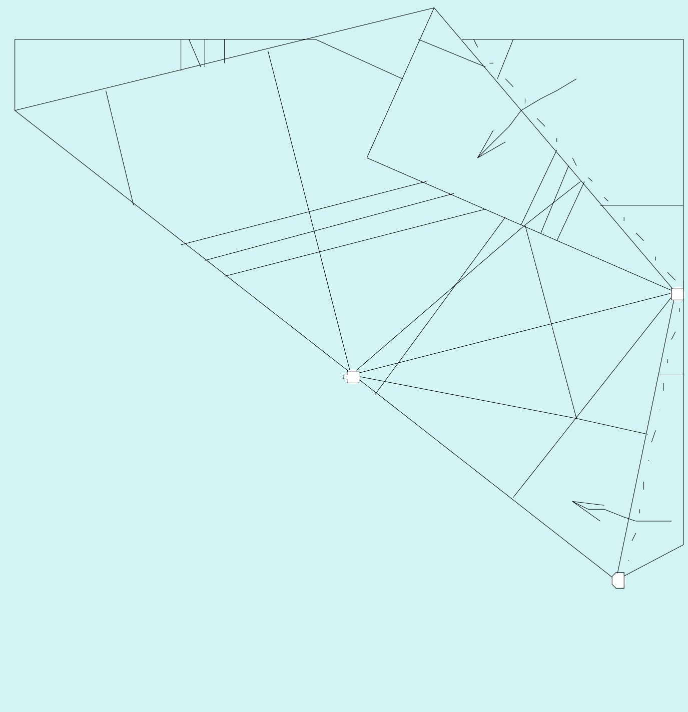

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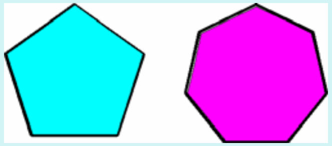

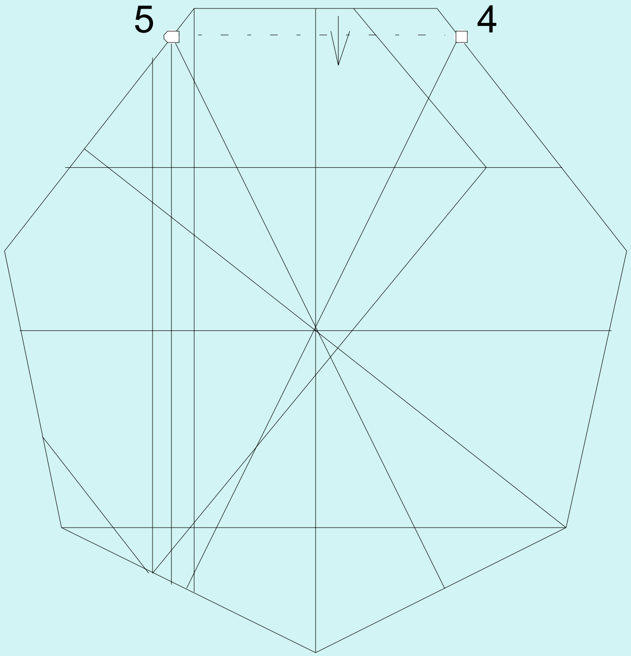

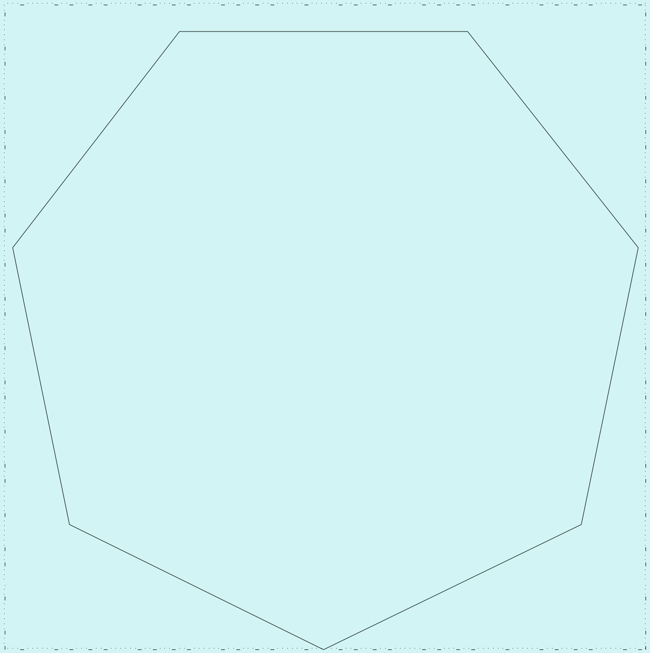

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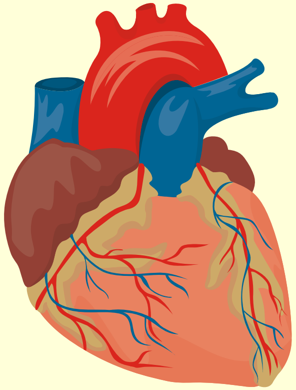

-lichen Dank für die Aufmerksamkeit